compunetix

MULTI-HYPOTHESIS MULTI-STAGE TRACKING APPLIED TO MARITIME SURVEILLANCE

Stefano Coraluppi

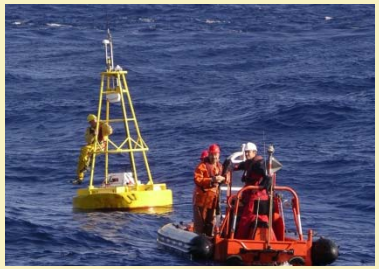
1.412.858.1746
1.412.373.7119 *fax*
stefano.coraluppi@compunetix.com

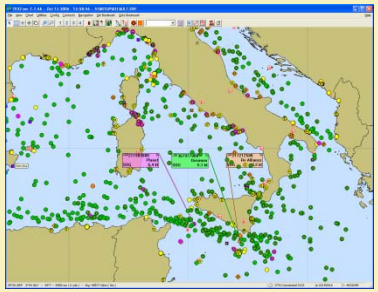
NATO RTO SET-157 Lecture Series, June 2011

WHY DATA FUSION?

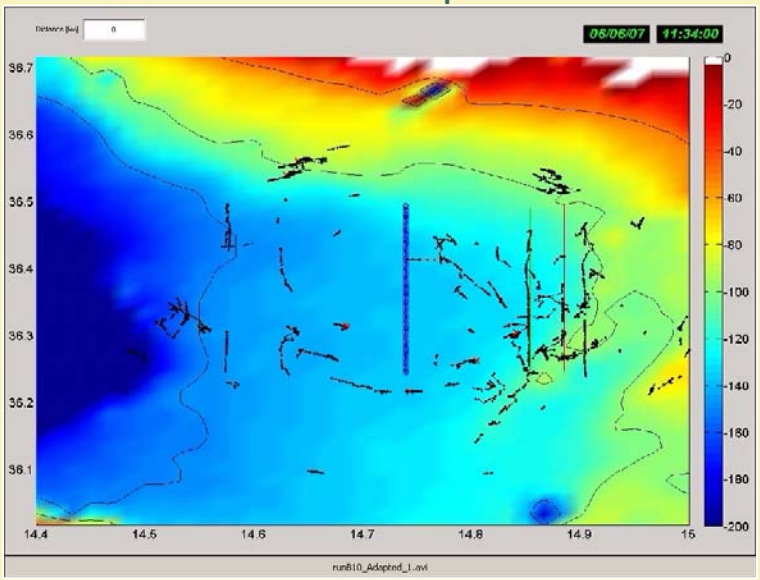
threats

sensors

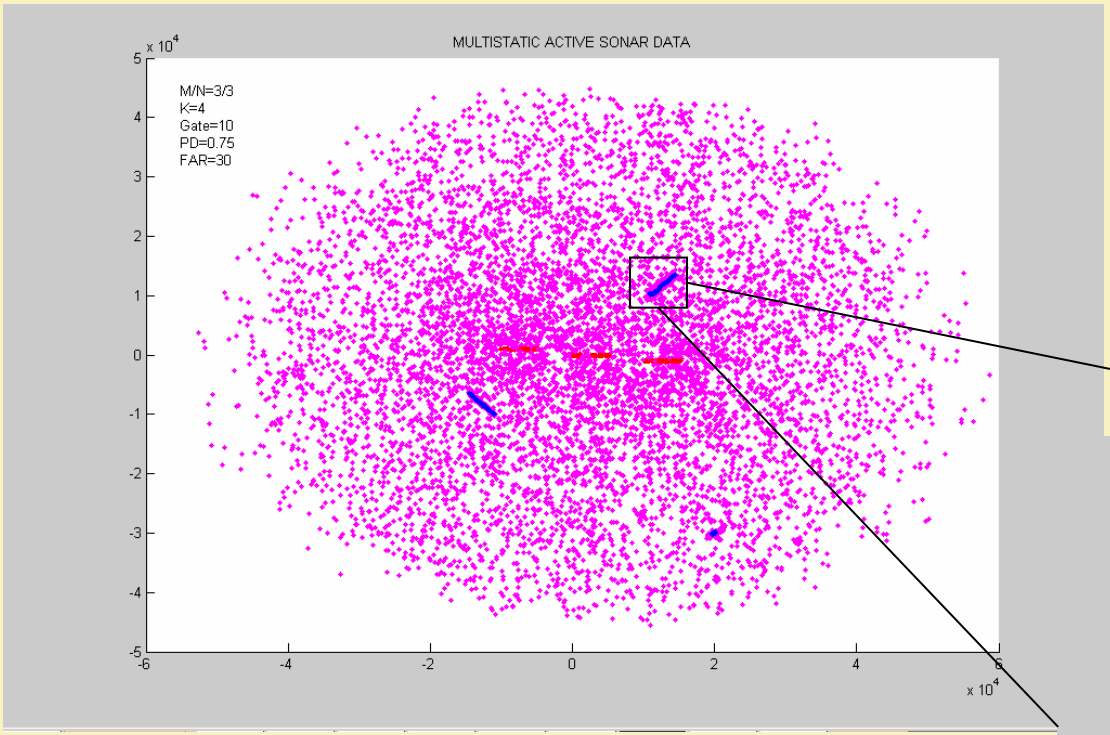


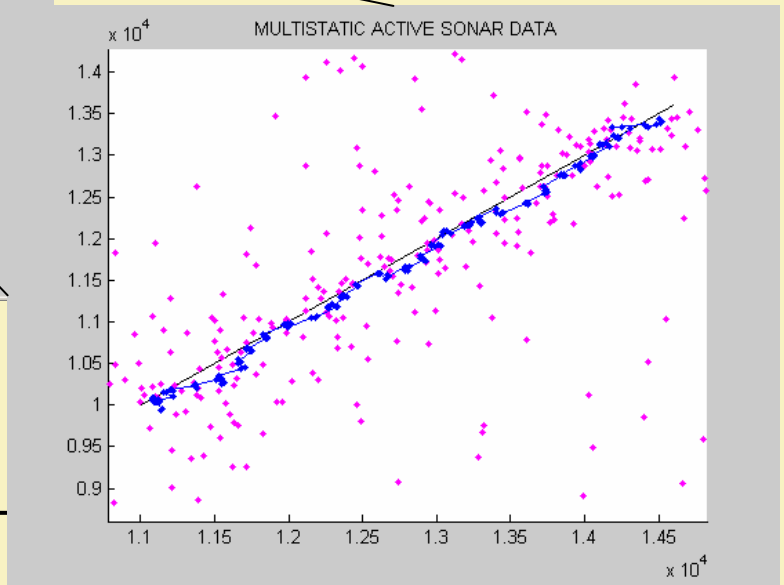


surveillance picture

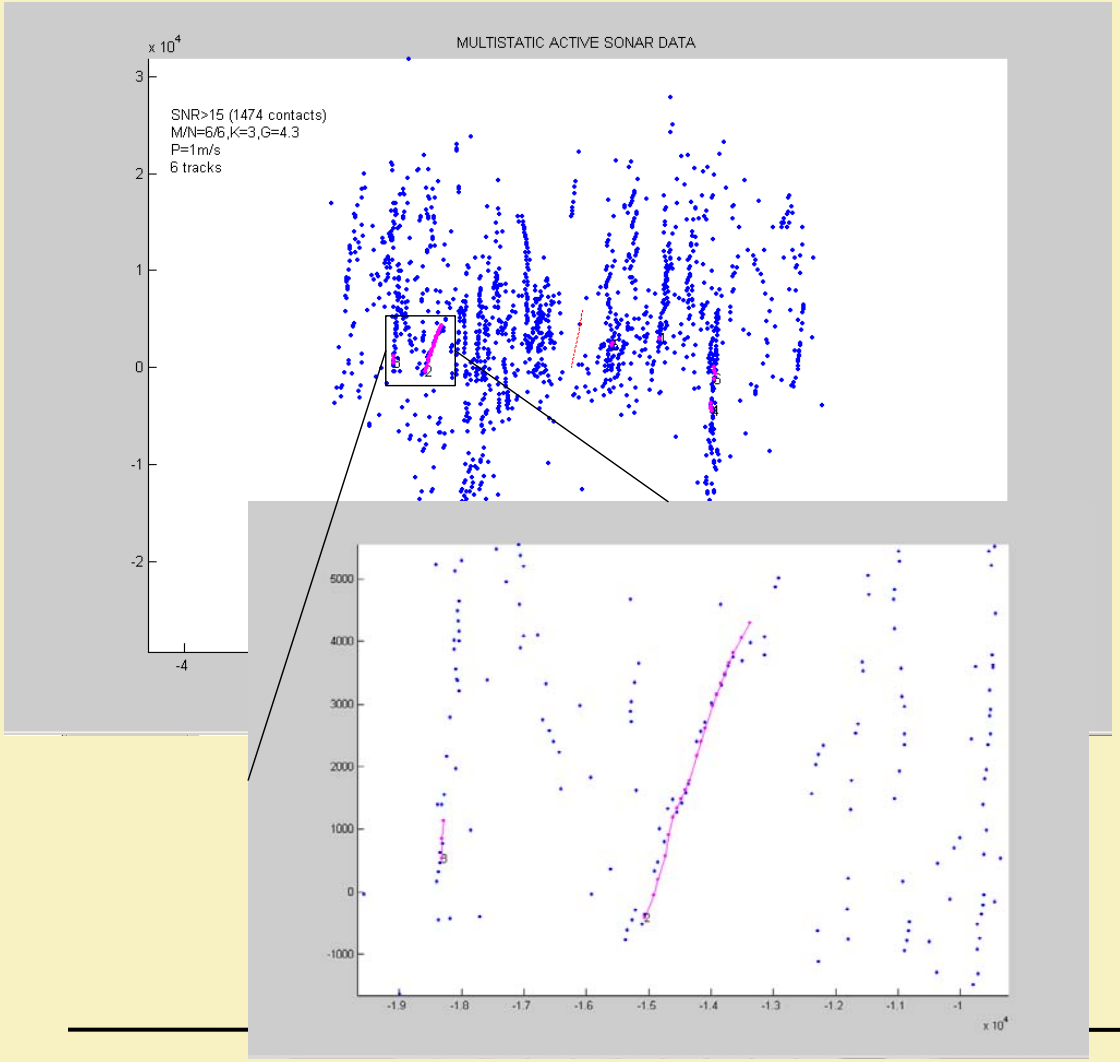


DATA FUSION ⇔ EXTRACTING A SIGNAL FROM NOISE





EXPERIMENTAL RESULTS – BACCHUS'98



1474 contacts with SNR>15
Key tracker parameters:
- track initiation:
  6 of 6 detections
- track termination:
  3 misses

6 tracks
(5 unknown, 1 sub)

OUTLINE

- **Target modeling**
- Bayesian filtering
- Multi-target filtering
- Multi-target tracking
- Cardinality tracking
- Distributed tracking
- Other topics

NEARLY CONSTANT POSITION (NCP) MODEL

- The model (in one dimension)

$$\dot{X}(t) = w(t)$$

$$X(\cdot) = x(\cdot)$$

$$E[w(t)] = 0$$

$$E[w(t)w(\tau)] = q\delta(t-\tau)$$

- Discretization

$$X_{k+1} = X_k + w_k \qquad w_k = \int_{t_k}^{t_{k+1}} w(t)dt \qquad E[w_k] = 0$$

$$E[w_k^2] = Q_k = \int_{t_k}^{t_{k+1}} w(t)dt \int_{t_k}^{t_{k+1}} w(\tau)d\tau$$

$$= \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} E[w(t)w(\tau)]d\tau dt = \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} q\delta(t-\tau)d\tau dt = \int_{t_k}^{t_{k+1}} q dt = q\Delta t_k$$

- Is this a good model?
  - Advantages: simple
  - Disadvantages: no velocity; unbounded position

NEARLY CONSTANT VELOCITY (NCV) MODEL

- The model

$$\dot{X}(t) = FX(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)$$

$$X(\cdot) = \begin{bmatrix} x(\cdot) \\ \dot{x}(\cdot) \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$E[w(t)] = 0$$

$$E[w(t)w(\tau)] = q\delta(t-\tau)$$

TIME DISCRETIZATION

- The *variation of constants* equation

$$\dot{X}(t) = FX(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)$$

$$X_{k+1} = A_k X_k + \int_{t_k}^{t_{k+1}} \exp\left(F\left(t_{k+1} - t\right)\right) \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)\,dt$$

$$A_k = \exp\left(F \Delta t_k\right)$$

$$\Delta t_k = t_{k+1} - t_k$$

SOLVING THE MATRIX EXPONENTIAL

- A numerical approach

$$\exp(Ft) = \sum_{k=0}^{\infty} \frac{t^k F^k}{k!}$$

- An analytical approach

$$\exp(Ft) = P \exp(\tilde{F}t) P^{-1}$$

$$\tilde{F} = P^{-1} F P$$

$$P = \begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix} \quad \tilde{F} = \begin{bmatrix} \Lambda_1 & 0 & \cdots & 0 \\ 0 & \Lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Lambda_n \end{bmatrix} \quad \Lambda_i = \begin{bmatrix} \lambda_i & 1 & \cdots & 0 \\ 0 & \lambda_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & \lambda_i \end{bmatrix}$$

$$\exp(\tilde{F}t) = \begin{bmatrix} \exp(\Lambda_1 t) & 0 & \cdots & 0 \\ 0 & \exp(\Lambda_2 t) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \exp(\Lambda_n t) \end{bmatrix} \quad \exp(\Lambda_i t) = \begin{bmatrix} \exp(\lambda_i t) & t\exp(\lambda_i t) & \cdots & \frac{t^{n-1}\exp(\lambda_i t)}{(n-1)!} \\ 0 & \exp(\lambda_i t) & \ddots & \vdots \\ \vdots & \ddots & \ddots & t\exp(\lambda_i t) \\ 0 & \cdots & 0 & \exp(\lambda_i t) \end{bmatrix}$$

DISCRETIZATION OF THE NCV MODEL

- Apply previous results to the NCV model

$$X_{k+1} = A_k X_k + \int_{t_k}^{t_{k+1}} \exp\left(F\left(t_{k+1} - t\right)\right)\begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)\,dt$$

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A_k = \exp\left(F\Delta t_k\right) = \begin{bmatrix} 1 & \Delta t_k \\ 0 & 1 \end{bmatrix}$$

$$X_{k+1} = A_k X_k + w_k$$

$$E\left[w_k\right] = 0 \qquad E\left[w_k w_k'\right] = Q_k = \begin{bmatrix} \dfrac{q\Delta t_k^3}{3} & \dfrac{q\Delta t_k^2}{2} \\ \dfrac{q\Delta t_k^2}{2} & q\Delta t_k \end{bmatrix}$$

DISCRETIZATION OF THE NCV MODEL

- The measurement noise covariance

$$E[w_k w_k'] = E\left[\int_{t_k}^{t_{k+1}} \begin{bmatrix} 1 & t_{k+1}-t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)dt \int_{t_k}^{t_{k+1}} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ t_{k+1}-\tau & 1 \end{bmatrix} w(\tau)d\tau\right]$$

$$= \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} E\left[\begin{bmatrix} t_{k+1}-t \\ 1 \end{bmatrix} \begin{bmatrix} t_{k+1}-\tau & 1 \end{bmatrix} w(t)w(\tau)dtd\tau\right]$$

$$= \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} \begin{bmatrix} (t_{k+1}-t)(t_{k+1}-\tau) & t_{k+1}-t \\ t_{k+1}-t & 1 \end{bmatrix} E[w(t)w(\tau)]dtd\tau$$

$$= \int_{t_k}^{t_{k+1}} \int_{t_k}^{t_{k+1}} \begin{bmatrix} (t_{k+1}-t)^2 & t_{k+1}-t \\ t_{k+1}-t & 1 \end{bmatrix} q\delta(t-\tau)d\tau dt$$

$$= \int_{t_k}^{t_{k+1}} \begin{bmatrix} (t_{k+1}-t)^2 & t_{k+1}-t \\ t_{k+1}-t & 1 \end{bmatrix} q dt$$

$$= \begin{bmatrix} \frac{q\Delta t_k^3}{3} & \frac{q\Delta t_k^2}{2} \\ \frac{q\Delta t_k^3}{2} & q\Delta t_k \end{bmatrix}$$

SUMMARY OF DISCRETE-TIME NCV MODEL

$$X_{k+1} = A_k X_k + w_k$$

$$E[w_k] = 0$$

$$A_k = \exp(F\Delta t_k) = \begin{bmatrix} 1 & \Delta t_k \\ 0 & 1 \end{bmatrix}$$

$$E[w_k w_k'] = Q_k = \begin{bmatrix} \frac{q\Delta t_k^3}{3} & \frac{q\Delta t_k^2}{2} \\ \frac{q\Delta t_k^2}{2} & q\Delta t_k \end{bmatrix}$$

- Is this a good model?
  - Advantages: simple; well-defined velocity
  - Disadvantages: unbounded position and velocity

ORNSTEIN-UHLENBECK (OU) PROCESS

- The model

$$\dot{X}(t) = -\gamma X(t) + w(t)$$

$$X(\cdot) = x(\cdot)$$

$$E[w(t)] = 0$$

$$E[w(t)w(\tau)] = q\delta(t-\tau)$$

- Discrete-time form

$$X_{k+1} = \exp(-\gamma \Delta t_k) X_k + w_k$$

$$E[w_k] = 0$$

$$E[w_k^2] = Q_k = q\frac{1-\exp(-2\gamma\Delta t_k)}{2\gamma}$$

- Is this a good model?
  - Advantages: bounded position
  - Disadvantages: no velocity

INTEGRATED ORNSTEIN-UHLENBECK (IOU) PROCESS

- The model

$$\dot{X}(t) = FX(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)$$

$$X(\cdot) = \begin{bmatrix} x(\cdot) \\ \dot{x}(\cdot) \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 1 \\ 0 & -\gamma \end{bmatrix}$$

$$E[w(t)] = 0$$

$$E[w(t)w(\tau)] = q\delta(t-\tau)$$

- Discrete-time form

$$X_{k+1} = A_k X_k + w_k$$

$$A_k = \begin{bmatrix} 1 & \dfrac{1-\exp(-\gamma\Delta t_k)}{\gamma} \\ 0 & \exp(-\gamma\Delta t_k) \end{bmatrix}$$

$$Q_k^{11} = \frac{q}{\gamma^2}\left\{\Delta t_k - 2\left(\frac{1-\exp(-\gamma\Delta t_k)}{\gamma}\right) + \frac{1}{2}\left(\frac{1-\exp(-2\gamma\Delta t_k)}{\gamma}\right)\right\}$$

$$E[w_k] = 0 \qquad E[w_k w_k'] = Q_k = \begin{bmatrix} Q_k^{11} & Q_k^{12} \\ Q_k^{21} & Q_k^{22} \end{bmatrix}$$

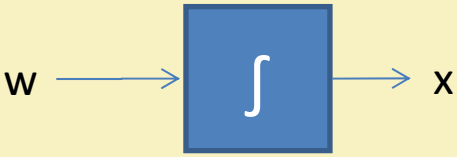
$$Q_k^{12} = Q_k^{21} = \frac{q}{\gamma^2}\left\{(1-\exp(-\gamma\Delta t_k)) - \frac{1}{2}(1-\exp(-2\gamma\Delta t_k))\right\}$$

$$Q_k^{22} = q\left(\frac{1-\exp(-2\gamma\Delta t_k)}{2\gamma}\right)$$

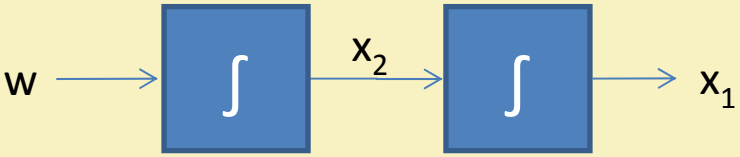
- Is this a good model?
  - Advantages: well-defined and bounded velocity
  - Disadvantages: unbounded position

AN ENGINEER'S VIEW

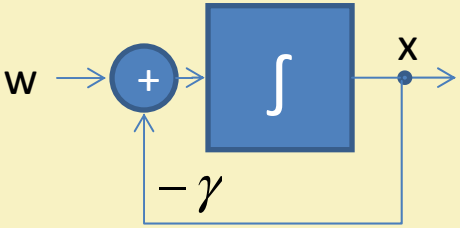
- Nearly constant position



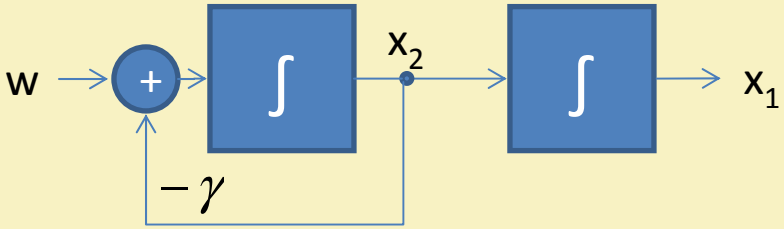
- Nearly constant velocity



- Ornstein Uhlenbeck

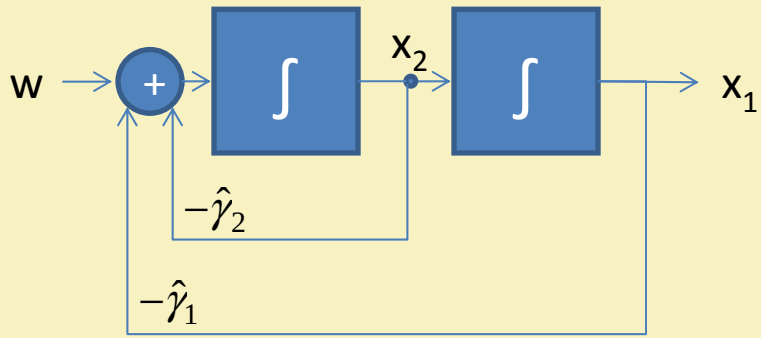


- Integrated Ornstein Uhlenbeck



MIXED ORNSTEIN-UHLENBECK (MOU) PROCESS

- We consider feedback in both position and velocity



- The model

$$\dot{X}(t) = FX(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w(t)$$

$$X(\cdot) = \begin{bmatrix} x(\cdot) \\ \dot{x}(\cdot) \end{bmatrix} \qquad F = \begin{bmatrix} 0 & 1 \\ -\hat{\gamma}_1 & -\hat{\gamma}_2 \end{bmatrix}$$

$$E[w(t)] = 0$$

$$E[w(t)w(\tau)] = q\delta(t-\tau)$$

DISCRETE-TIME MOU PROCESS

$$X_{k+1} = A_k\left(X_k - \bar{X}\right) + w_k + \bar{X}$$

$$\bar{X} = \begin{bmatrix} \bar{x} \\ 0 \end{bmatrix}$$

$$\gamma_1, \gamma_2 = \frac{\hat{\gamma}_2 \pm \sqrt{\hat{\gamma}_2^2 - 4\hat{\gamma}_1}}{2}$$

$$A_k = \frac{1}{\gamma_1 - \gamma_2}\begin{bmatrix} -\gamma_2 \exp(-\gamma_1 \Delta t_k) + \gamma_1 \exp(-\gamma_2 \Delta t_k) & -\exp(-\gamma_1 \Delta t_k) + \exp(-\gamma_2 \Delta t_k) \\ \gamma_1\gamma_2 \exp(-\gamma_1 \Delta t_k) - \gamma_1\gamma_2 \exp(-\gamma_2 \Delta t_k) & \gamma_1 \exp(-\gamma_1 \Delta t_k) - \gamma_2 \exp(-\gamma_2 \Delta t_k) \end{bmatrix}$$

$$E[w_k] = 0$$

$$E[w_k w_k'] = Q_k = \begin{bmatrix} Q_k^{11} & Q_k^{12} \\ Q_k^{12} & Q_k^{22} \end{bmatrix}$$

$$Q_k^{11} = \frac{q}{(\gamma_1 - \gamma_2)^2}\left\{\left(\frac{1 - \exp(-2\gamma_1 \Delta t_k)}{2\gamma_1}\right) + \left(\frac{1 - \exp(-2\gamma_2 \Delta t_k)}{2\gamma_2}\right) - 2\left(\frac{1 - \exp(-(\gamma_1 + \gamma_2)\Delta t_k)}{\gamma_1 + \gamma_2}\right)\right\}$$

$$Q_k^{12} = \frac{q}{2(\gamma_1 - \gamma_2)^2}\left\{\exp(-2\gamma_1 \Delta t_k) + \exp(-2\gamma_2 \Delta t_k) - 2\exp(-(\gamma_1 + \gamma_2)\Delta t_k)\right\}$$

$$Q_k^{22} = \frac{q}{(\gamma_1 - \gamma_2)^2}\left(\frac{\gamma_1}{2}(1 - \exp(-2\gamma_1 \Delta t_k)) + \frac{\gamma_2}{2}(1 - \exp(-2\gamma_2 \Delta t_k)) - \frac{2\gamma_1\gamma_2}{\gamma_1 + \gamma_2}(1 - \exp(-(\gamma_1 + \gamma_2)\Delta t_k))\right)$$

- Is this a good model?
  - Advantages: well-defined and bounded velocity; bounded position; well-defined stationary form

THE STATIONARY MOU PROCESS

- For stationarity, initial target statistics must obey limiting form

$$X_{k+1} = A_k\left(X_k - \overline{X}\right) + w_k + \overline{X}$$

$$\Delta t_k \to \infty$$

$$A_k \to 0 \qquad Q_k \to \overline{Q} = q\begin{bmatrix} \dfrac{1}{2\hat{\gamma}_1\hat{\gamma}_2} & 0 \\ 0 & \dfrac{1}{2\hat{\gamma}_2} \end{bmatrix}$$

$$X_0 \sim N\left(\overline{X}, \overline{Q}\right)$$

- We can express stationary form in terms of quantities of interest

$$\overline{Q} = \begin{bmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}$$

$$\hat{\gamma}_2 = \frac{q}{2\sigma_v^2}$$

$$\hat{\gamma}_1 = \frac{\sigma_v^2}{\sigma_p^2}$$

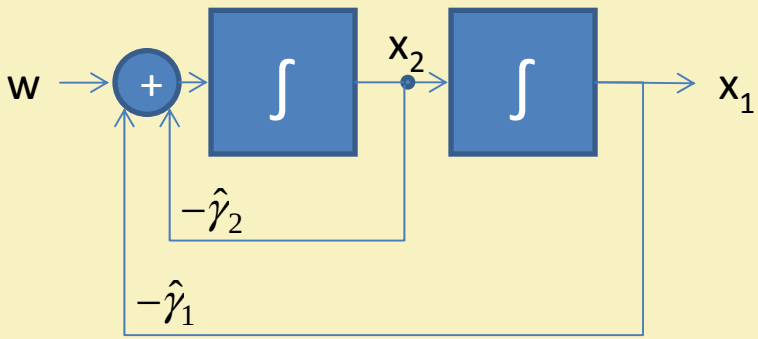
$$\hat{\gamma}_2^2 - 4\hat{\gamma}_1 > 0 \Rightarrow \sigma_p > \frac{4\sigma_v^3}{q}$$

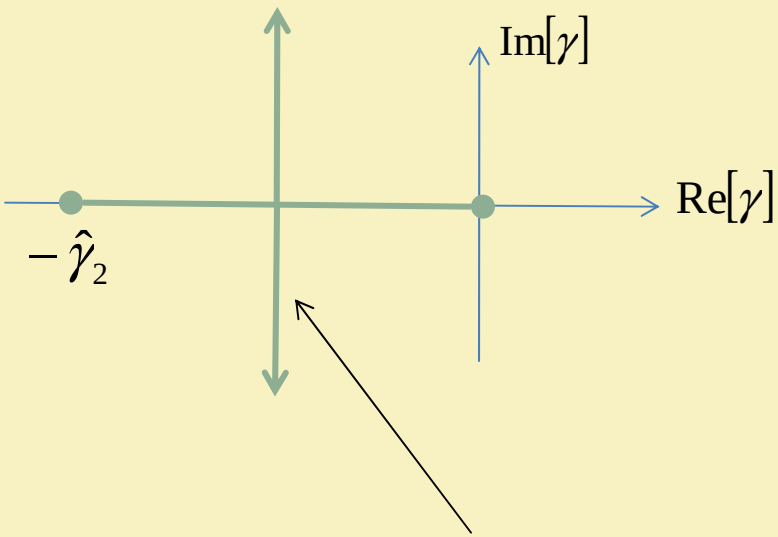
A CONNECTION TO CONTROL THEORY

- The root locus view is useful

$$\gamma_1, \gamma_2 = \frac{\hat{\gamma}_2 \pm \sqrt{\hat{\gamma}_2^2 - 4\hat{\gamma}_1}}{2}$$

$$\hat{\gamma}_1 = \gamma_1 \gamma_2 \qquad \hat{\gamma}_2 = \gamma_1 + \gamma_2$$





path followed by eigenvalues for increasing $\hat{\gamma}_1$

- For stability, we require $\hat{\gamma}_1, \hat{\gamma}_2 > 0 \Leftrightarrow \gamma_1, \gamma_2 > 0$

REPEATED AND COMPLEX EIGENVALUES

$$X_{k+1} = A_k\left(X_k - \overline{X}\right) + w_k + \overline{X} \qquad X_0 \sim N\left(\overline{X}, \overline{Q}\right)$$

$$\overline{Q} = q\begin{bmatrix} \dfrac{1}{2\hat{\gamma}_1\hat{\gamma}_2} & 0 \\ 0 & \dfrac{1}{2\hat{\gamma}_2} \end{bmatrix} = \begin{bmatrix} \sigma_p^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}$$

- Repeated eigenvalues $\gamma = \gamma_1 = \gamma_2$

$$A_k = \begin{bmatrix} (1+\gamma\Delta t_k)\exp(-\gamma\Delta t_k) & \Delta t_k \exp(-\gamma\Delta t_k) \\ -\gamma^2\Delta t_k \exp(-\gamma\Delta t_k) & (1-\gamma\Delta t_k)\exp(-\gamma\Delta t_k) \end{bmatrix} \qquad Q_k^{12} = \frac{q}{2}\Delta t_k^2 \exp(-2\gamma\Delta t_k)$$

$$Q_k^{11} = \frac{q}{8\gamma^3}\left(2 - \left(4\gamma^2\Delta t_k^2 + 4\gamma\Delta t_k + 2\right)\exp(-2\gamma\Delta t_k)\right) \qquad Q_k^{22} = \frac{q}{4\gamma}\left(1 - \exp(-2\gamma\Delta t_k)\right) + \frac{q\Delta t_k}{2}\exp(-2\gamma\Delta t_k)(1-\gamma\Delta t_k)$$

- Complex eigenvalues

$$\gamma_1 = \alpha + j\beta = \frac{\hat{\gamma}_2 + \sqrt{\hat{\gamma}_2^2 - 4\hat{\gamma}_1}}{2}$$

$$\gamma_2 = \alpha - j\beta = \frac{\hat{\gamma}_2 - \sqrt{\hat{\gamma}_2^2 - 4\hat{\gamma}_1}}{2}$$

$$A_k = \exp(-\alpha\Delta t_k)\begin{bmatrix} \cos(\beta\Delta t_k) + \dfrac{\alpha}{\beta}\sin(\beta\Delta t_k) & \dfrac{1}{\beta}\sin(\beta\Delta t_k) \\ -\dfrac{\alpha^2+\beta^2}{\beta}\sin(\beta\Delta t_k) & \cos(\beta\Delta t_k) - \dfrac{\alpha}{\beta}\sin(\beta\Delta t_k) \end{bmatrix}$$

$$Q_k^{11} = \frac{q}{4\alpha\beta^2\left(\alpha^2+\beta^2\right)}\left\{\beta^2 + \exp(-2\alpha\Delta t_k)\left(\alpha^2\cos(2\beta\Delta t_k) - \alpha\beta\sin(2\beta\Delta t_k) - \alpha^2 - \beta^2\right)\right\}$$

$$Q_k^{12} = \frac{q\exp(-2\alpha\Delta t_k)}{4\beta^2}\left(1 - \cos(2\beta\Delta t_k)\right)$$

$$Q_k^{22} = \frac{q}{4\alpha}\left(1 - \exp(-2\alpha\Delta t_k)\right) + \frac{q\exp(-2\alpha\Delta t_k)}{4\beta^2}\left(\alpha\cos(2\beta\Delta t_k) + \beta\sin(2\beta\Delta t_k) - \alpha\right)$$

MULTIPLE TARGETS

- Poisson process
  - Birth rate λ_b
  - Death rate λ_χ

- Discrete time
  - Birth rate $\lambda_{b,k+1} = \int_{t_k}^{t_{k+1}} \lambda_b \cdot \exp\left(-\lambda_\chi (t_{k+1} - t)\right) dt = \frac{\lambda_b}{\lambda_\chi}\left(1 - \exp\left(-\lambda_\chi \Delta t_k\right)\right)$

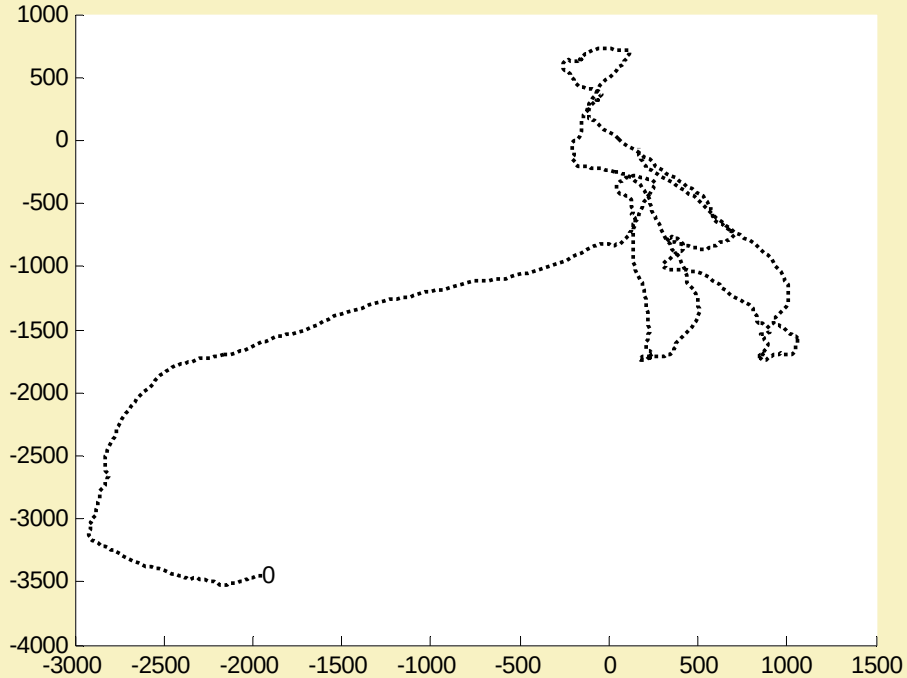
    $\lambda_\chi \to 0 \Rightarrow \lambda_{b,k+1} \to \lambda_b \Delta t_k$

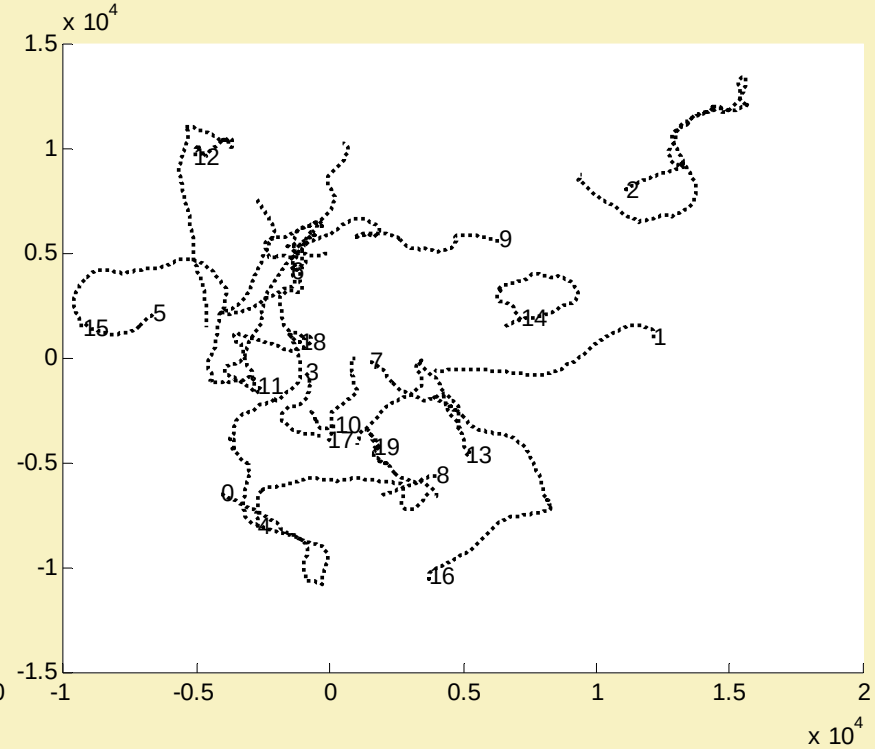
  - Death probability $p_{\chi,k+1} = \int_{t_k}^{t_{k+1}} \lambda_\chi \exp\left(-\lambda_\chi t\right) dt = 1 - \exp\left(-\lambda_\chi \Delta t_k\right).$

- Stationary Poisson point process $\Delta t_k \to \infty \Rightarrow \lambda_{b,k+1} \to \frac{\lambda_b}{\lambda_\chi}$
  - Birth rate at first scan $\lambda_{b,1} = \frac{\lambda_b}{\lambda_\chi}$
  - Target state at birth $X \sim N\left(\overline{X}, \overline{Q}\right)$

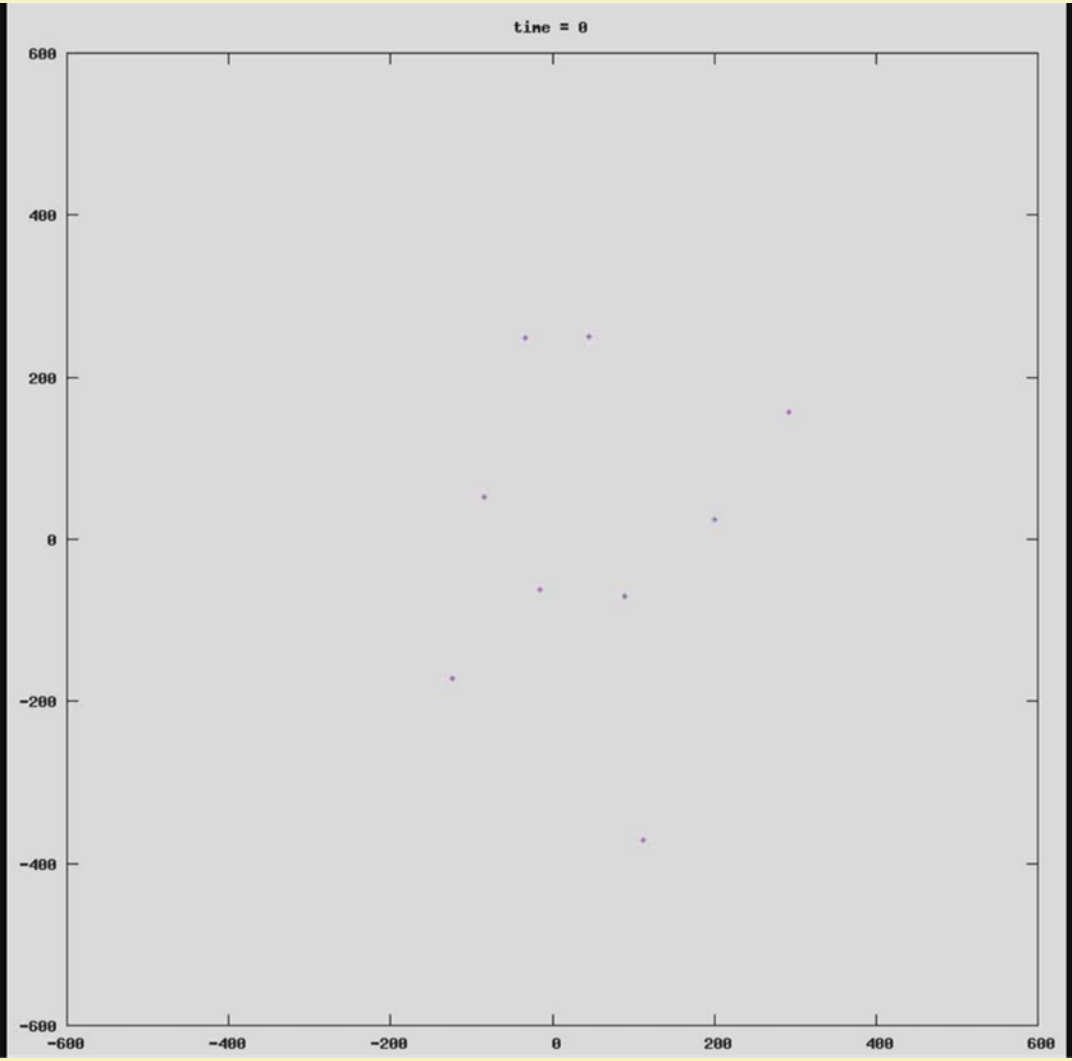
ILLUSTRATIONS

- Single-target realization (left), stationary MOU Poisson point process (right)



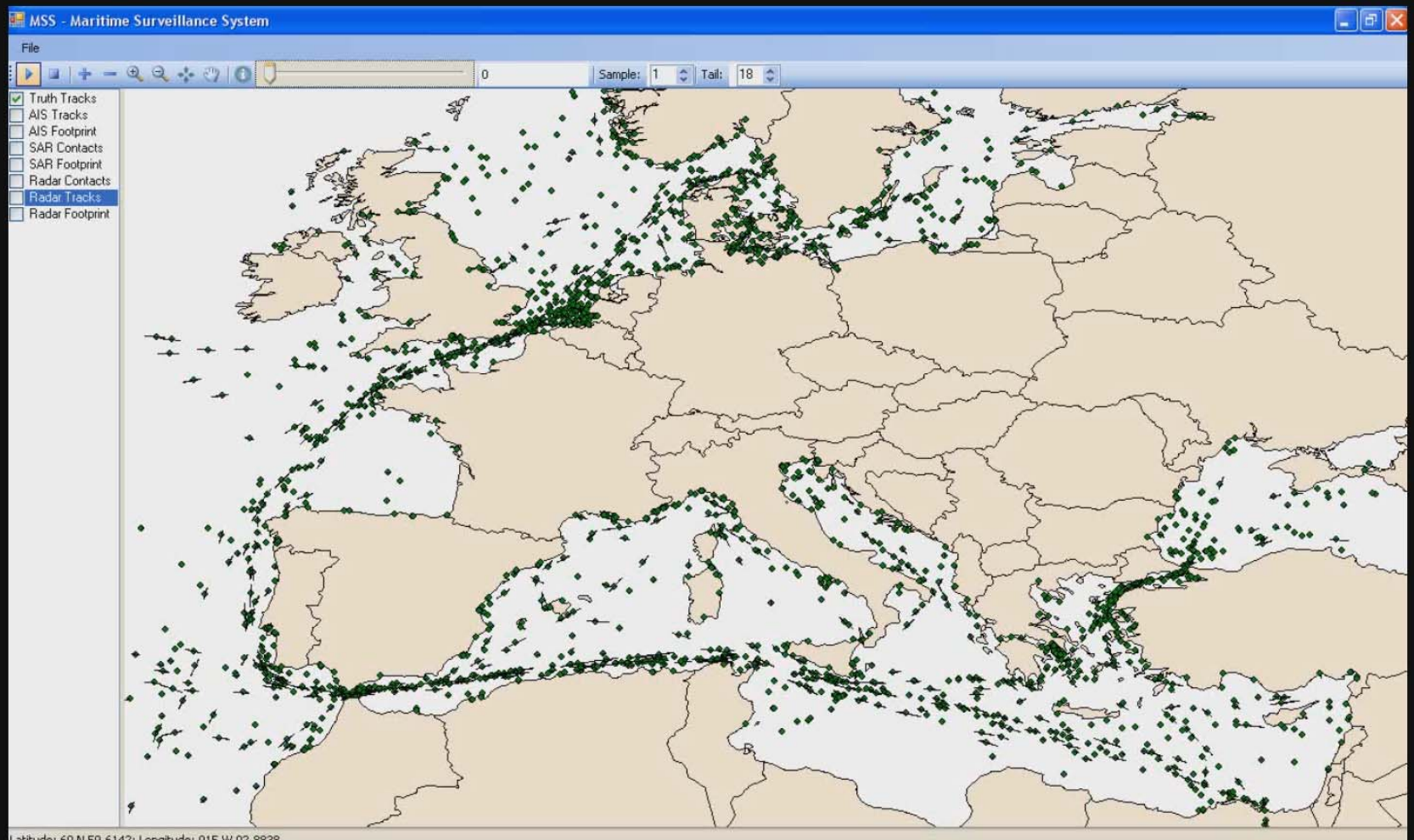


STOCHASTIC MOTION EXAMPLE



compunetix

DETERMINISTIC MOTION EXAMPLE



OUTLINE

- Target modeling
- **Bayesian filtering**
- Multi-target filtering
- Multi-target tracking
- Cardinality tracking
- Distributed tracking
- Other topics

BAYESIAN ESTIMATION

- Bayes risk

$$J = E_X\left[E_Y\left[C\left(\hat{X}(Y), X\right) \mid X\right]\right]$$
$$= E_{X,Y}\left[C\left(\hat{X}(Y), X\right)\right] = E_Y\left[E_X\left[C\left(\hat{X}(Y), X\right) \mid Y\right]\right]$$

  - Estimator $\hat{X}(Y)$
  - Cost $C\left(\hat{X}(Y), X\right)$

- *Minimum mean squared error* (MMSE) estimator is the conditional mean

$$C(x,y) = (x-y)^2 \Rightarrow \hat{X}(y) = E[X \mid y]$$

- *Minimum mean absolute error* (MMAE) estimator is the conditional median

$$C(x,y) = |x-y| \Rightarrow \hat{X}(y) = M[X \mid y]$$

- *Maximum a posteriori* (MAP) estimator is the conditional mode

$$C(x,y) = 1 \Leftrightarrow |x-y| > \Delta \Rightarrow \hat{X}(y) = \arg\max f(X \mid y)$$
$$= \arg\max f(y \mid X) f(X)$$

AN EXAMPLE

- A priori distribution $f(x) = \alpha \exp(-\alpha x), x \geq 0$ $f(y \mid x) = x \exp(-xy), y \geq 0$
- Conditional distribution $$f(x \mid y) = \frac{f(y \mid x) f(x)}{\int_0^\infty f(y \mid x) f(x) dx} = (\alpha + y)^2 x \exp(-x(\alpha + y))$$
- Estimators
  - A priori $\hat{X}_{MMSE} = \frac{1}{\alpha}$ $\hat{X}_{MMAE} = \frac{\log 2}{\alpha}$ $\hat{X}_{MAP} = 0$
  - A posteriori $\hat{X}_{MMSE}(y) = \frac{2}{\alpha + y}$ $\hat{X}_{MMAE}(y) = \frac{\beta}{\alpha + y}$ $\hat{X}_{MAP}(y) = \frac{1}{\alpha + y}$

    $(1 + \beta)\exp(-\beta) = \frac{1}{2} \Rightarrow \beta \approx 1.68$
- Points to note
  - MMSE, MMAE, and MAP estimators differ in general
  - Computation of the MAP estimator does not require the unconditional density $f(y)$

$f(y \mid x) f(x) = \alpha x \exp(-x(\alpha + y))$ $0 = \frac{\partial [f(y \mid x) f(x)]}{\partial x} = (\alpha - x(\alpha + y))\exp(-x(\alpha + y)) \Rightarrow x = \frac{1}{\alpha + y}$

LEAST SQUARES ESTIMATION

- Random vectors $x \in R^n$ $y \in R^m$ $\begin{bmatrix} x \\ y \end{bmatrix} \sim D\left(\begin{bmatrix} \bar{x} \\ \bar{y} \end{bmatrix}, \begin{bmatrix} \Sigma_x & \Sigma_{xy} \\ \Sigma_{yx} & \Sigma_y \end{bmatrix} \right)$
- *Estimator* $x(\cdot): R^m \to R^n$
- *Least squares estimator* minimizes $\underset{x,y}{E}\left\{ \| x - x(y) \|^2 \right\}$ $\|z\|^2 = z'z$
- Theorem 1: least squares estimator (LSE) is given by $x^*(y) = \underset{x}{E}\{x \mid y\}$ for all *y*
- Theorem 2: linear least squares estimator (LLSE)

$$\hat{x}(y) = \bar{x} + \Sigma_{xy}\Sigma_y^{-1}(y - \bar{y}) \qquad \underset{x,y}{E}\left\{ (x - \hat{x}(y))(x - \hat{x}(y))' \right\} = \Sigma_x - \Sigma_{xy}\Sigma_y^{-1}\Sigma_{yx}$$

- Theorem 3: in linear case, LLSE is recursive (Kalman filter)
- Theorem 4: in linear Gaussian case, Kalman filter is the LSE

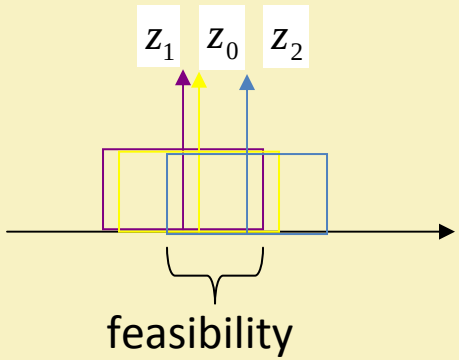
AN EXAMPLE

- Unknown scalar *x*, to be estimated based on a sequence of measurements

$$z_k = x + v_k, k = 0,\ldots n-1 \qquad v_k \sim U\left(-\tfrac{1}{2}, \tfrac{1}{2}\right)$$

- Kalman filter in batch form

$$x_{n-1|n-1} = \frac{(z_0 + z_1 + \ldots + z_{n-1})}{n} \qquad \sigma^2_{n-1|n-1} = \frac{\sigma_v^2}{n} = \frac{1}{12n}$$



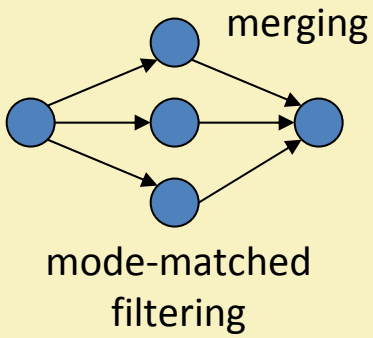
- Least squares estimator

$$x^*\left(z^{n-1}\right) = \frac{\max\{z_0,\ldots,z_{n-1}\} + \min\{z_0,\ldots,z_{n-1}\}}{2} \qquad \sigma^2_{x^*} = \frac{1}{2(n+1)(n+2)}$$

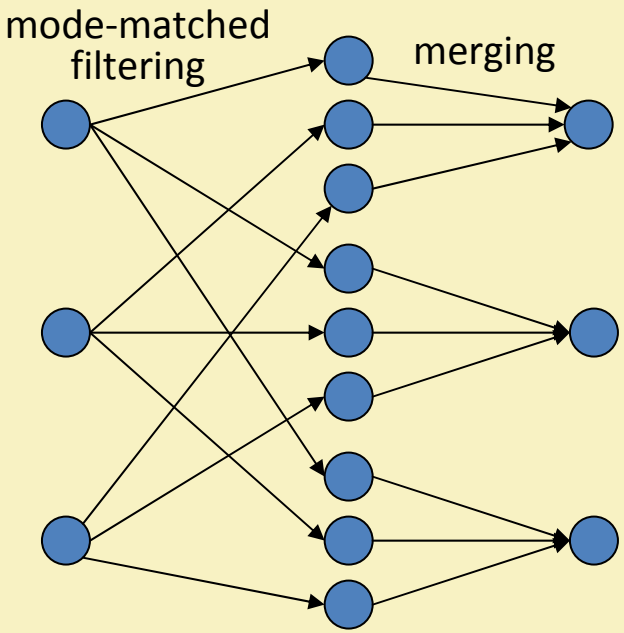
- Solution is recursive but not linear
- Kalman filter is not optimal

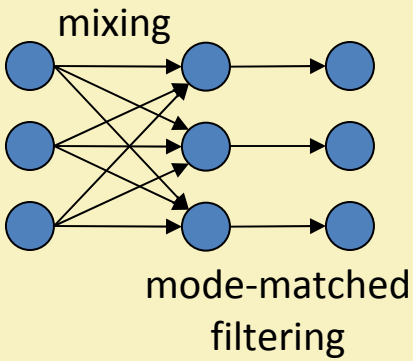
MULTIPLE-MODEL FILTERING

- Generalized Pseudo-Bayesian (GPB)
  - Level 1: *N* filters needed



  - Level 2: N^2 filters needed
- Interacting Multiple Model (IMM)
  - *N* filters needed
  - Similar performance to GPB2



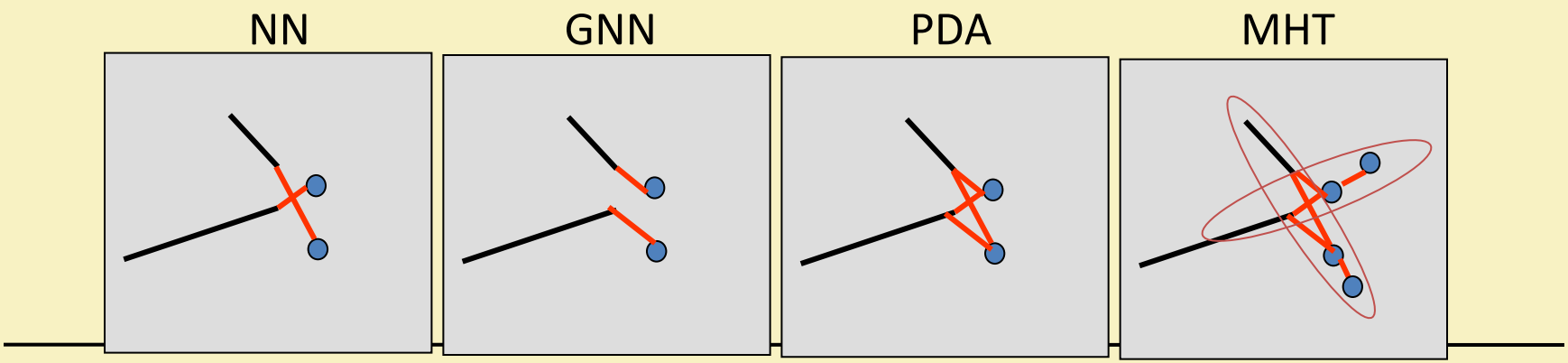


OUTLINE

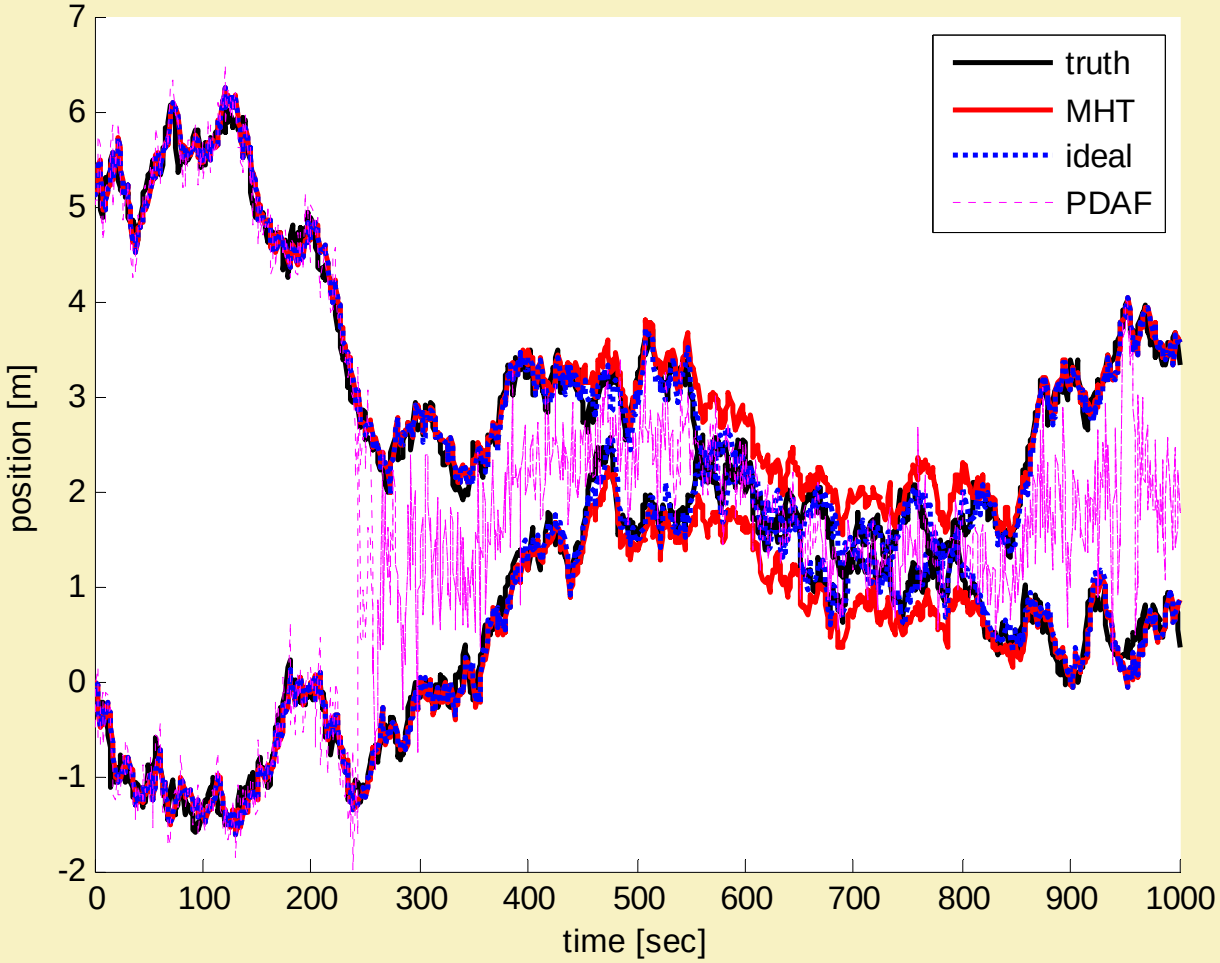
- Target modeling
- Bayesian filtering
- **Multi-target filtering**
- Multi-target tracking
- Cardinality tracking
- Distributed tracking
- Other topics

MULTI-TARGET FILTERS

- Nearest neighbor (NN)
  - Munkres algorithm (exact)
  - Auction algorithm (suboptimal, fast)
- Global nearest neighbor (GNN)
- Probabilistic data association (PDA)
  - Extensions include Joint PDA (JPDA)
- Multiple hypothesis tracking (MHT)
  - Hypothesis oriented
  - Track oriented



COALESCENCE AND REPULSION



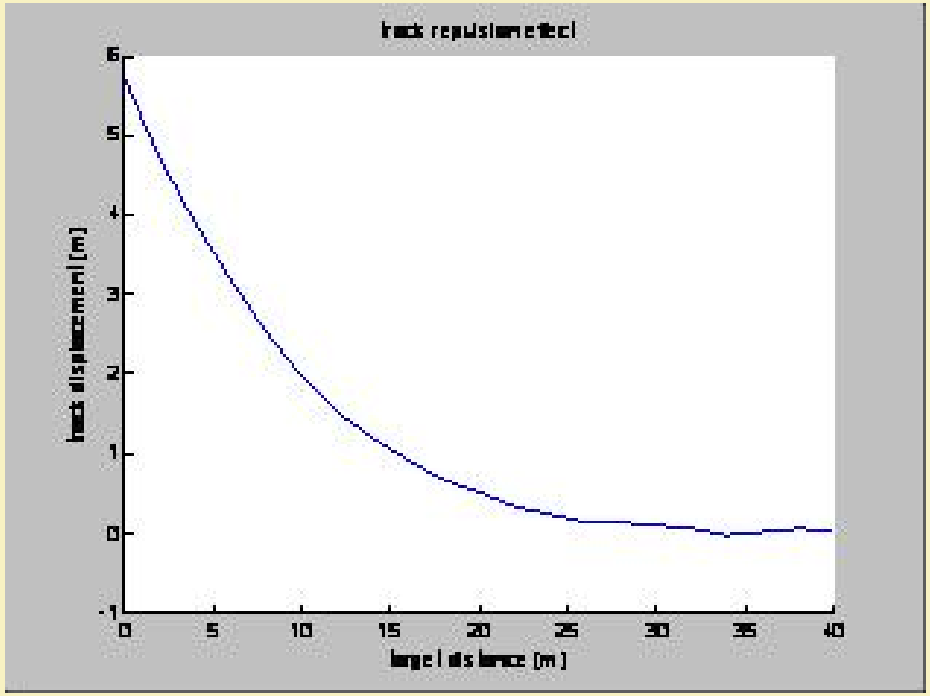
ANALYSIS

$$Y_k^{(i)} = X_k^{(i)} + v_k^{(i)},\ k = 1,\ldots,N$$

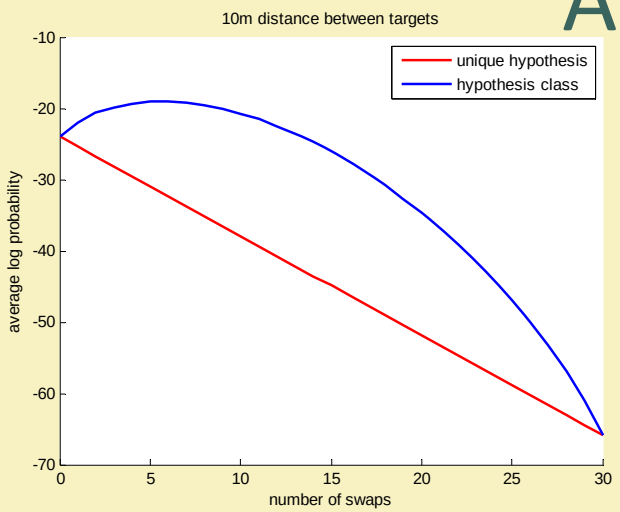
$$\hat{X}^{(1)} = -\hat{X}^{(2)} = \overline{X}$$

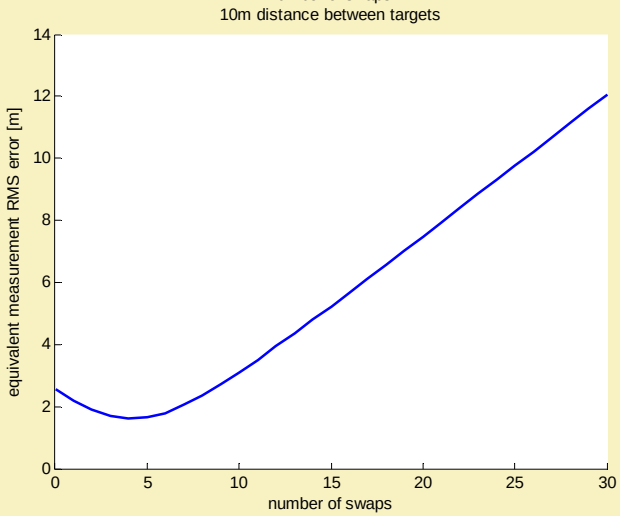
$$X_k^{(1)} = -X_k^{(2)} = \overline{X},\ k = 1,\ldots,N$$

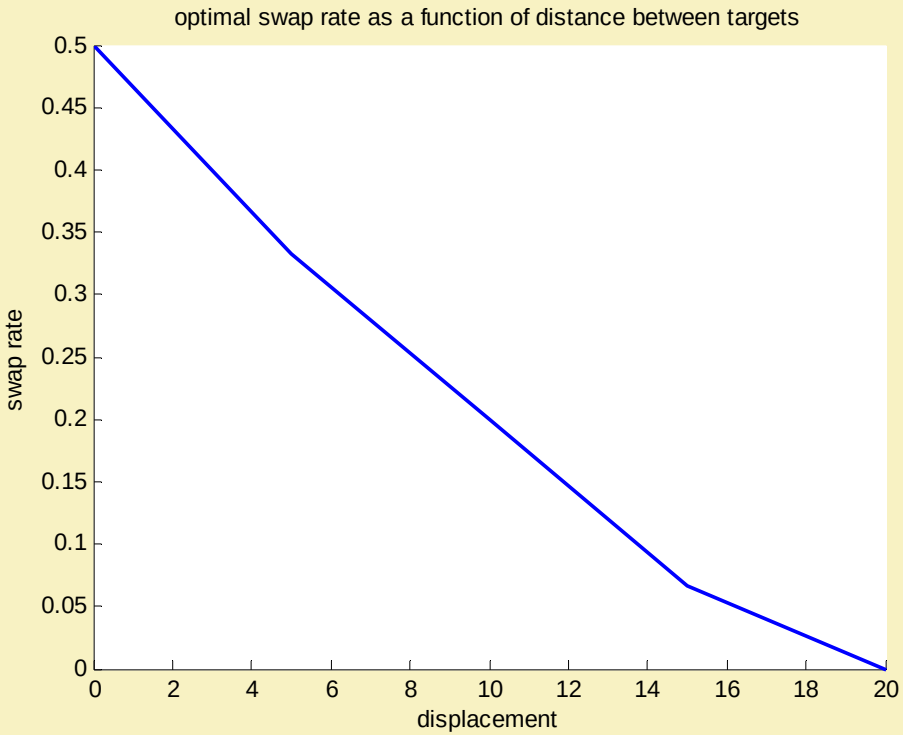
$$\hat{X}^{(1)} = -\hat{X}^{(2)} = E\left[\max\left\{\overline{X} + v, -\overline{X} + w\right\}\right]$$



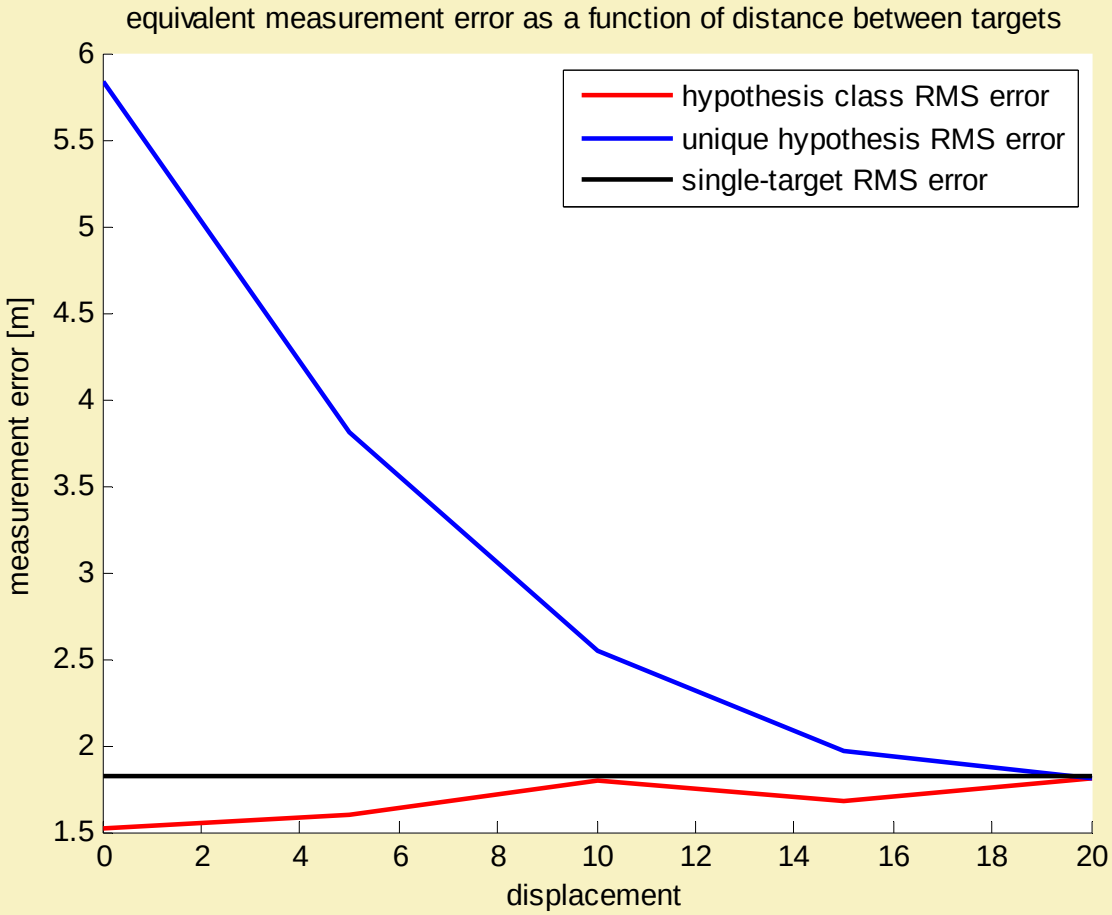
A NEW EQUIVALENCE-CLASS MHT APPROACH (1)



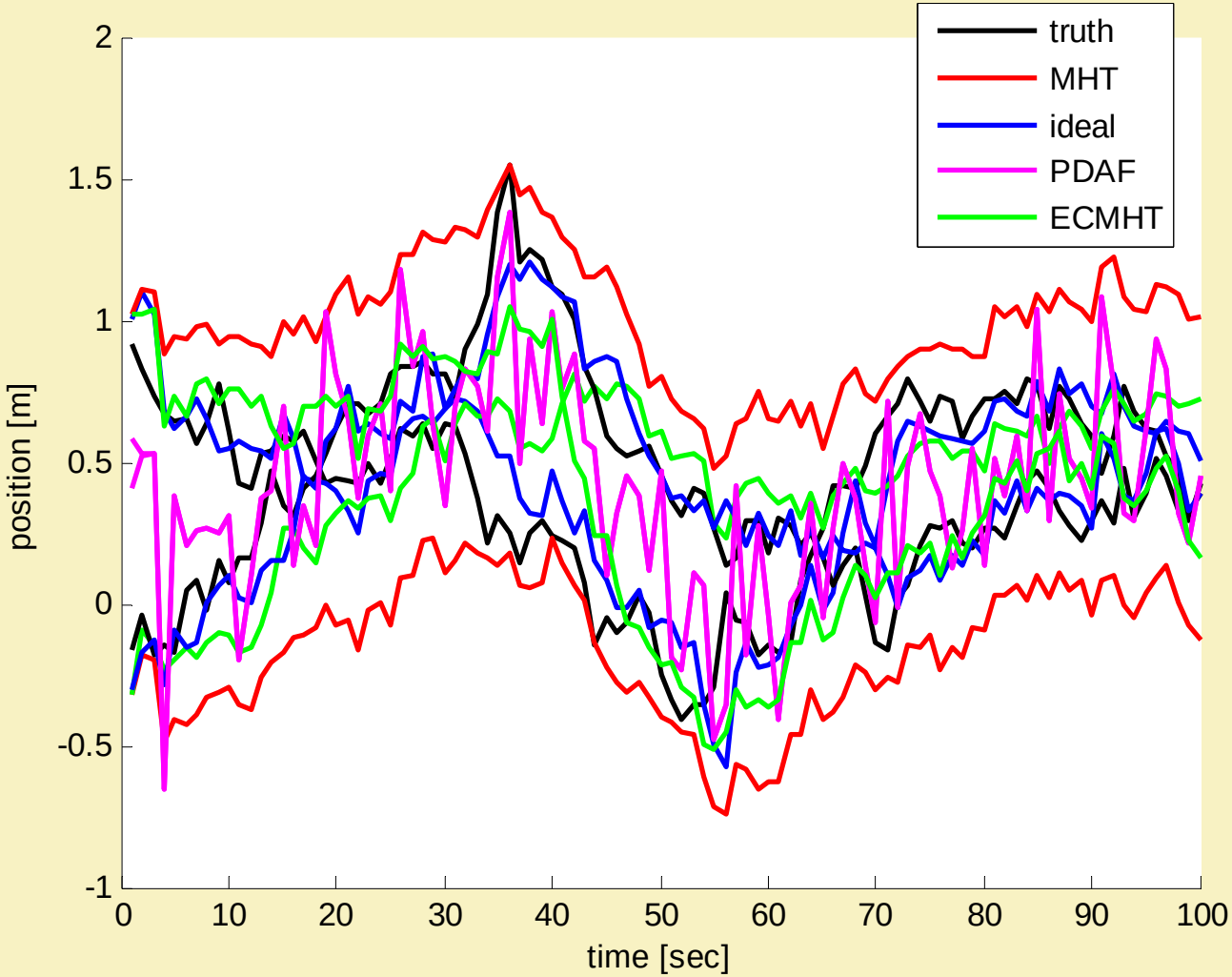




A NEW EQUIVALENCE-CLASS MHT APPROACH (2)



AN ILLUSTRATION



SIMULATION DETAILS

- 1D OU motion model (two targets)

$$X_{k+1} = A_k X_k + w_k \quad X_0 \sim N\left(0, \frac{q}{2\gamma}\right)$$

$$A_k = \exp(-\gamma \Delta t_k) \quad \gamma > 0 \quad \Delta t_k = t_{k+1} - t_k$$

$$E[w_k] = 0 \quad E\left[w_k^2\right] = Q_k = \frac{1 - \exp(-2\gamma \Delta t_k)}{2\gamma} q$$

- Measurements (no missed detections, no false alarms)

$$Z_k = C_k X_k + v_k \quad E[v_k] = 0 \quad E\left[v_k^2\right] = R_k$$

$$(\{Z_{11}, Z_{12}\}, \{Z_{21}, Z_{22}\}, \{Z_{31}, Z_{32}\}, \ldots)$$

$$Z_{k1} \le Z_{k2}$$

INITIALIZATION AND PREDICTION

- Initialization

$$X_1(1|1) = Z_{11} \quad X_2(1|1) = Z_{12}$$

$$P_j(1|1) = R_1,\ j = 1,2$$

- Prediction

$$X_j(k+1|k) = A_k X_j(k|k),\ j = 1,2$$

$$P_j(k+1|k) = A_k P_j(k|k) A_k' + Q_k$$

$$S_j(k+1) = C_{k+1} P_j(k+1|k) C_{k+1}' + R_{k+1}$$

$$L_j(k+1) = P_j(k+1|k) C_{k+1}' S_j^{-1}(k+1),\ j = 1,2$$

PROBABILISTIC DATA ASSOCIATION FILTER (PDAF)

$$\beta_{ij}(k+1|k) = c_j \exp\left(-\frac{\left(Z_{ij} - C_k X_j(k+1|k)\right)^2}{2S_j(k+1)}\right), i = 1,2,\ j = 1,2 \qquad c_j = 1 \Big/ \sum_i \beta_{ij}(k+1|k)$$

$$\upsilon_{ij}(k+1) = Z_{ij} - C_{k+1} X(k+1|k), i = 1,2,\ j = 1,2$$

$$\upsilon_j(k+1) = \sum_i \beta_{ij} \upsilon_{ij}(k+1),\ j = 1,2$$

$$X_j(k+1|k+1) = X_j(k+1|k) + L_j(k+1)\upsilon_j(k+1),\ j = 1,2$$

$$P_j(k+1|k+1) = \left(I - L_j(k+1)C_{k+1}\right)P_j(k+1|k) + \tilde{P}_j(k+1),\ j = 1,2$$

$$\tilde{P}_j(k+1) = L_j(k+1)\left(\sum_i \beta_{ij}\upsilon_{ij}^2(k+1) - \upsilon_j^2(k+1)\right)L_j(k+1),\ j = 1,2$$

MULTIPLE-HYPOTHESIS TRACKING (MHT)

$$\upsilon_1(k+1) = Z_{(k+1)1} - X_1(k+1 \mid k)$$

$$\upsilon_2(k+1) = Z_{(k+1)2} - X_2(k+1 \mid k)$$

$$X_j(k+1 \mid k+1) = X_j(k+1 \mid k) + L_j(k+1)\upsilon_j(k+1),\ j = 1, 2$$

$$P(k+1 \mid k+1) = (I - L(k+1)C_{k+1})P(k+1 \mid k)$$

EQUIVALENCE-CLASS MHT

$$X_1(1|1) = \sum_n Z_{11}(n) \qquad P_j(1|1) = \frac{R_1}{N},\ j = 1,2$$

$$X_2(1|1) = \sum_n Z_{12}(n) \qquad S_j(k+1) = C_{k+1} P_j(k+1|k) C'_{k+1} + \frac{R_{k+1}}{N}$$

$$p\left(\left\{q_i^k\right\} | Z^k\right) \approx \binom{k}{i} p\left(q_i^k | Z^k\right)$$

$$\omega(k+1) = \arg\max_s \left\{ \binom{N}{s} \left(\begin{array}{l} \sum_{i=1}^{s} \begin{pmatrix} -\left(Z_{(k+1)1}(i) - X_1(k+1|k)\right)^2 \\ -\left(Z_{(k+1)2}(i) - X_2(k+1|k)\right)^2 \end{pmatrix} \\ + \sum_{i=s+1}^{N} \begin{pmatrix} -\left(Z_{(k+1)2}(i) - X_1(k+1|k)\right)^2 \\ -\left(Z_{(k+1)1}(i) - X_2(k+1|k)\right)^2 \end{pmatrix} \end{array} \right) \right\}$$

$$Z_{(k+1)1} = \frac{\sum_{i=1}^{\omega(k+1)} Z_{(k+1)1}(i) + \sum_{i=\omega(k+1)+1}^{N} Z_{(k+1)2}(i)}{N}$$

$$Z_{(k+1)2} = \frac{\sum_{i=1}^{\omega(k+1)} Z_{(k+1)2}(i) + \sum_{i=\omega(k+1)+1}^{N} Z_{(k+1)1}(i)}{N}$$

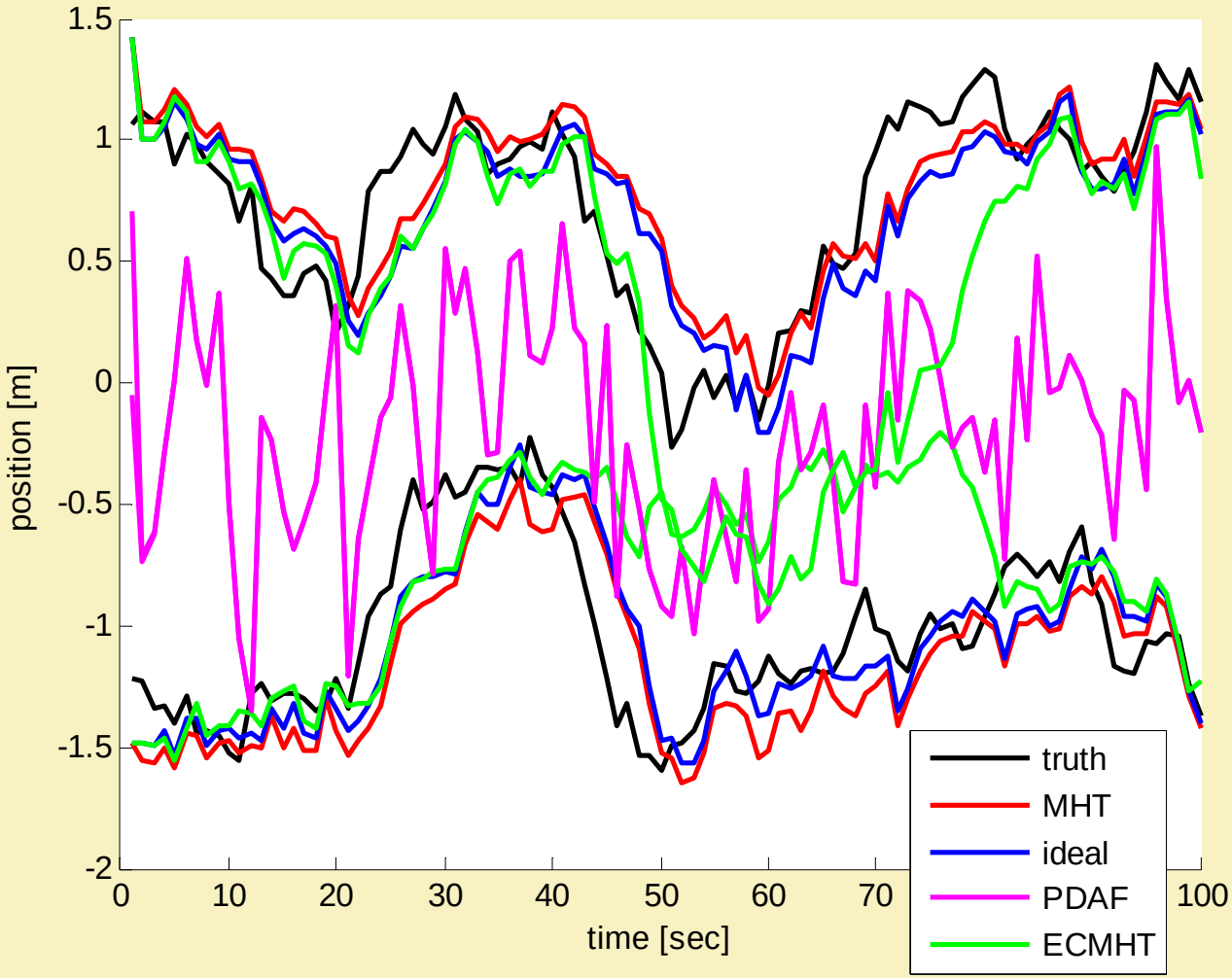
$$\upsilon_1(k+1) = Z_{(k+1)1} - X_1(k+1|k)$$

$$\upsilon_2(k+1) = Z_{(k+1)2} - X_2(k+1|k)$$

$$X_j(k+1|k+1) = X_j(k+1|k) + L_j(k+1)\upsilon_j(k+1),\ j = 1,2$$

$$P(k+1|k+1) = \left(I - L(k+1)C_{k+1}\right)P(k+1|k)$$

ANOTHER REALIZATION



PERFORMANCE NUMBERS

$q = 0.01\text{m}^2\text{sec}^{-1}$

$\gamma = 0.05$

$\Delta t = 1\text{sec}$

$R = 1\text{m}^2$

$N = 10$

100 Monte Carlo realizations

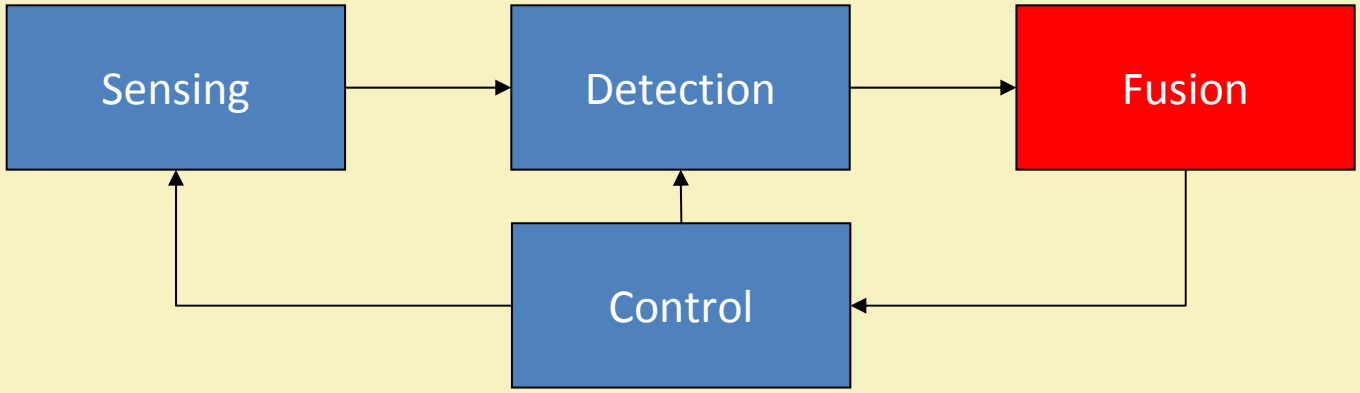
Filtering scheme	**MOSPA**
ideal	0.0309m
PDAF	1.4637m
MHT	0.1433m
ECMHT	0.0784m

OUTLINE

- Target modeling
- Bayesian filtering
- Multi-target filtering
- **Multi-target tracking**
- Cardinality tracking
- Distributed tracking
- Other topics

DATA FUSION PARADIGMS

- Batch vs. scan-based processing
- Unified detection & tracking vs. contact-based tracking
- Soft vs. hard data association
- Analytical vs. Numerical filtering
- Labelled vs. unlabelled tracking
- Low-level vs. high-level fusion
- Centralized vs. distributed processing



MULTI-TARGET TRACKING: HYBRID-STATE FORMULATION

- The tracking problem
  - Given a sequence of sets of measurements Z^k
  - Wish to estimate state history of all targets assuming *at most* one contact per target per scan X^k
- A useful decomposition
  - Interpretation for all target existence states and data associations q^k

$$
\begin{aligned}
p\left(X^k \mid Z^k\right) &= \sum_{q^k} p\left(X^k, q^k \mid Z^k\right) \\
&= \sum_{q^k} p\left(X^k \mid Z^k, q^k\right) p\left(q^k \mid Z^k\right)
\end{aligned}
$$

47

POSTERIOR DISTRIBUTION FOR DISCRETE STATE (1)

- Objective is a recursive and computationally-efficient expression for the discrete state probability distribution $p\left(q^k \mid Z^k\right)$
  - Bayesian decomposition & conditioning step

$$p\left(q^k \mid Z^k\right) = \frac{p\left(Z_k \mid Z^{k-1}, q^k\right) p\left(q^k \mid Z^{k-1}\right)}{c_k}$$

$$= \frac{p\left(Z_k \mid Z^{k-1}, q^k\right) p\left(q_k \mid Z^{k-1}, q^{k-1}\right) p\left(q^{k-1} \mid Z^{k-1}\right)}{c_k}$$

$$c_k = p\left(Z_k \mid Z^{k-1}\right) = \sum_{q^k} p\left(Z_k \mid Z^{k-1}, q^k\right) p\left(q^k \mid Z^{k-1}\right)$$ normalizing constant

  - Aggregate state variable ψ_k

$$p\left(q_k \mid Z^{k-1}, q^{k-1}\right) = p\left(\psi_k \mid Z^{k-1}, q^{k-1}\right) p\left(q_k \mid Z^{k-1}, q^{k-1}, \psi_k\right)$$

  - The aggregate state identifies the numbers of target births b, target deaths χ, target detections d, and reports r

POSTERIOR DISTRIBUTION FOR DISCRETE STATE (2)

- Aggregate state variable term

$$p\left(\psi_k \mid Z^{k-1}, q^{k-1}\right) = \left\{\binom{\tau}{\chi} p_\chi^\chi \left(1-p_\chi\right)^{\tau-\chi}\right\}\left\{\binom{\tau-\chi}{d} p_d^d \left(1-p_d\right)^{\tau-\chi-d}\right\}\left\{\frac{\exp\left(-\lambda_b\right)\lambda_b^b}{b!}\right\}\left\{\frac{\exp\left(-\lambda_{fa}\right)\lambda_{fa}^{r-d-b}}{(r-d-b)!}\right\}$$

- Term with aggregate state variable conditioning

$$p\left(q_k \mid Z^{k-1}, q^{k-1}, \psi_k\right) = \frac{1}{\binom{\tau}{\chi}\binom{\tau-\chi}{d}\left(\frac{r!}{(r-d)!}\right)\binom{r-d}{b}}$$

- Overall discrete state variable term

$$p\left(q_k \mid Z^{k-1}, q^{k-1}\right) = \left\{\frac{\exp\left(-\lambda_b-\lambda_{fa}\right)\lambda_{fa}^r}{r!}\right\} p_\chi^\chi \left(\left(1-p_\chi\right)\left(1-p_d\right)\right)^{\tau-\chi-d}\left(\frac{\left(1-p_\chi\right)p_d}{\lambda_{fa}}\right)^d\left(\frac{\lambda_b}{\lambda_{fa}}\right)^b$$

- Continuous state variable term

$$p\left(Z_k \mid Z^{k-1}, q^k\right) = \prod_{j\in J_d} f_d\left(z_j \mid Z^{k-1}, q^k\right)\prod_{j\in J_b} f_b\left(z_j \mid Z^{k-1}, q^k\right)\prod_{j\in J_{fa}} f_{fa}\left(z_j \mid Z^{k-1}, q^k\right)$$

POSTERIOR DISTRIBUTION FOR DISCRETE STATE (3)

- Final *global hypothesis probability* recursive expression

$$p\left(q^k \mid Z^k\right) = p_\chi^\chi \left(\left(1-p_\chi\right)\left(1-p_d\right)\right)^{\tau-\chi-d} \prod_{j\in J_d}\left[\frac{\left(1-p_\chi\right)p_d\, f_d\left(z_j \mid Z^{k-1}, q^k\right)}{\lambda_{fa}\, f_{fa}\left(z_j \mid Z^{k-1}, q^k\right)}\right] \prod_{j\in J_b}\left[\frac{\lambda_b\, f_b\left(z_j \mid Z^{k-1}, q^k\right)}{\lambda_{fa}\, f_{fa}\left(z_j \mid Z^{k-1}, q^k\right)}\right] \frac{p\left(q^{k-1} \mid Z^{k-1}\right)}{\bar{c}_k}$$

deaths | missed detections | detections | births | prior & normalizing constant

- Normalizing constant

$$\bar{c}_k = \frac{c_k}{\left\{\frac{\exp\left(-\lambda_b - \lambda_{fa}\right)}{r!}\lambda_{fa}^r\right\} \prod_{j\in J_d \cup J_b \cup J_{fa}} f_{fa}\left(z_j \mid Z^{k-1}, q^k\right)}$$

- Variation: replace λ_b by $p_d \cdot \lambda_b$ (in either case, we do not reason over *undetected* births)

SO WHAT IS MHT?

- The optimal Bayesian tracker

$$p\left(X^k \mid Z^k\right) = \sum_{q^k} p\left(X^k, q^k \mid Z^k\right) = \sum_{q^k} p\left(X^k \mid Z^k, q^k\right) p\left(q^k \mid Z^k\right)$$

- A reasonable solution to the tracking problem

$$\hat{X}_{MMSE}\left(Z^k\right) = E\left[X^k \mid Z^k\right] = \sum_{q^k} E\left[X^k \mid Z^k, q^k\right] p\left(q^k \mid Z^k\right)$$

- A sub-optimal approach, known as multiple-hypothesis tracking (MHT)
  - Identify (with some delay) a single discrete state

$$\hat{q}^k = \hat{q}_{MAP}\left(Z^k\right) = \arg\max p\left(q^k \mid Z^k\right)$$

  - Conditioned on the discrete state, identify the continuous state

$$\hat{X}\left(Z^k\right) = \hat{X}_{MMSE}\left(Z^k, \hat{q}^k\right)$$

MAKING MHT FEASIBLE & REAL-TIME

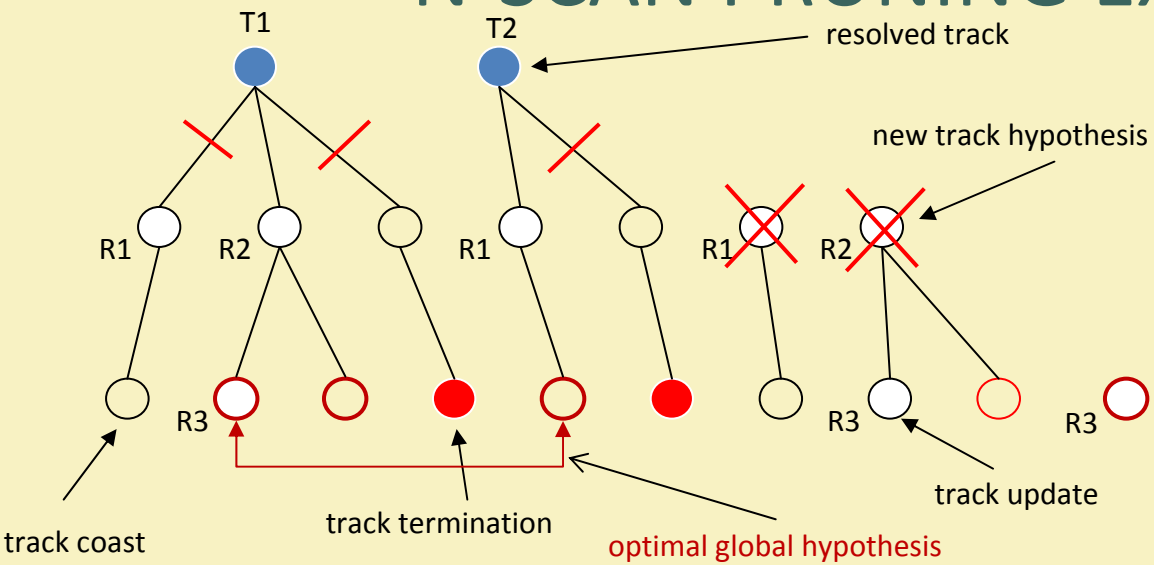
- The posterior distribution factors – intuition leading to track-oriented MHT

$$\hat{q}^k = \hat{q}_{MAP}\left(Z^k\right) = \arg\max p\left(q^k \mid Z^k\right)$$

$$p\left(q^k \mid Z^k\right) = \underbrace{p_\chi^\chi}_{\text{deaths}} \underbrace{\left(\left(1-p_\chi\right)\left(1-p_d\right)\right)^{\tau-\chi-d}}_{\text{missed detections}} \underbrace{\prod_{j\in J_d}\left[\frac{\left(1-p_\chi\right)p_d\, f_d\left(z_j \mid Z^{k-1}, q^k\right)}{\lambda_{fa}\, f_{fa}\left(z_j \mid Z^{k-1}, q^k\right)}\right]}_{\text{detections}} \underbrace{\prod_{j\in J_b}\left[\frac{\lambda_b\, f_b\left(z_j \mid Z^{k-1}, q^k\right)}{\lambda_{fa}\, f_{fa}\left(z_j \mid Z^{k-1}, q^k\right)}\right]}_{\text{births}} \underbrace{\frac{p\left(q^{k-1} \mid Z^{k-1}\right)}{\bar{c}_k}}_{\text{prior \& normalizing constant}}$$

- Sequential limitation in growth of local (track) hypotheses
  - Examples: include only one of *track termination* and *track coast; a*ccount for all contact data in target hypotheses; start tracks on detections
- Sequential removal of some track hypotheses
  - Example: *n-scan* processing based on the *estimated* MAP or *maximum likelihood* (ML) solution
- Track extraction via concurrent, sequential processing
  - Examples: *sequential probability ratio test* (SPRT) and logic-based schemes

N-SCAN PRUNING EXAMPLE



objective $\max c'x$

subject to $Ax = b$

$x_i \in [0,1]$

(for each vector element)

$$b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Account for tracks T1 and T2

Account for reports R1, R2, and R3

Track hypotheses

$$\left.\begin{array}{l} \max c'x \\ Ax = b \\ x \in [0,1]^N \end{array}\right\} \Rightarrow \tilde{x} \Rightarrow \left\{\begin{array}{l} \max \tilde{x}'x \\ Ax = b \\ x \in \{0,1\}^N \end{array}\right.$$

DOES MHT USE MAP OR ML PROCESSING?

- Relationship between MAP and ML estimators
  - Recall the definition of the MAP estimator

$$J = E_X\left[E_Y\left[C\left(\hat{X}(Y), X\right) \mid X\right]\right]$$
$$= E_{X,Y}\left[C\left(\hat{X}(Y), X\right)\right] = E_Y\left[E_X\left[C\left(\hat{X}(Y), X\right) \mid Y\right]\right]$$

$$C(x, y) = 1 \Leftrightarrow |x - y| > \Delta \Rightarrow \hat{X}(y) = \arg\max f(X \mid y)$$
$$= \arg\max f(y \mid X) f(X)$$

  - ML estimator is *equivalent* to MAP with uniform prior

$$\hat{X}(y) = \arg\max f(y \mid X)$$

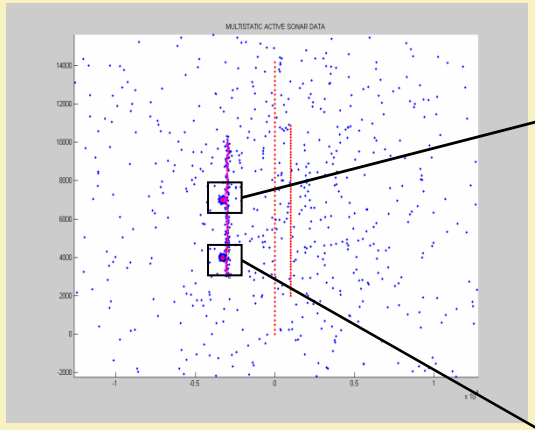
  - The ML estimator is a *non-Bayesian* estimator
  - In our context, MAP and ML approaches are the same, as there is a single (prior) global hypothesis

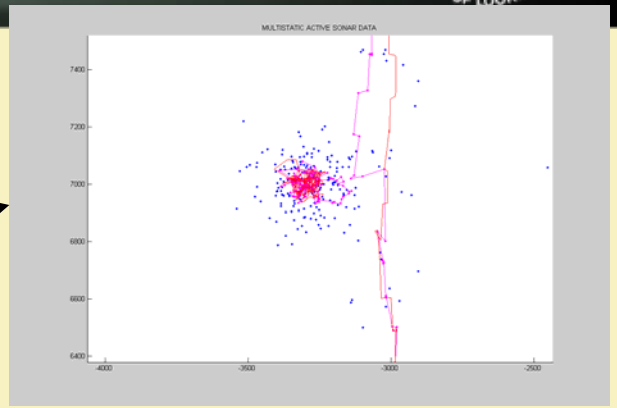
ISSUES

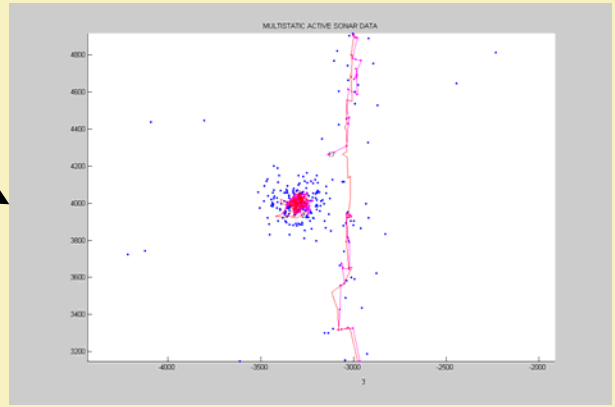
- Why do we need track confirmation and termination criteria?
  - A downstream threshold test to reduce spurious tracks
  - A means to invalidate associations that involve degenerate tracks
  - A scheme that works on a sliding-window basis so as to avoid batch track extraction
- Is it better to use equality or inequality constraints in MAP estimation step?
- Should MHT be modified to prefer greedy data association to favor confirmed tracks?
- How should we score track birth hypotheses in large-scale problems (with varying scan rates and sensor footprints)?

SIMULATION-BASED TESTING

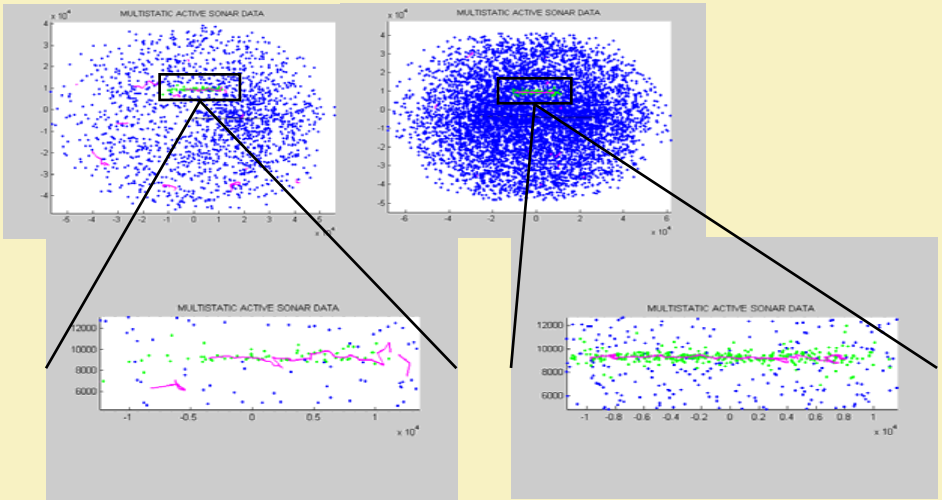
- MHT performance

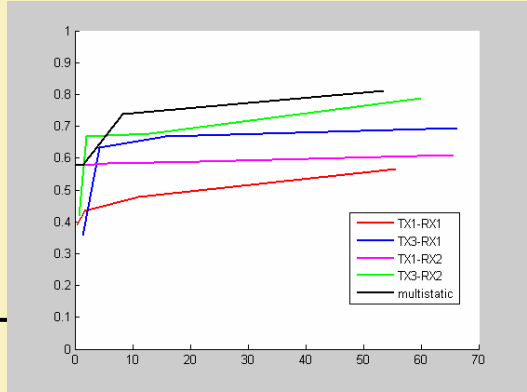






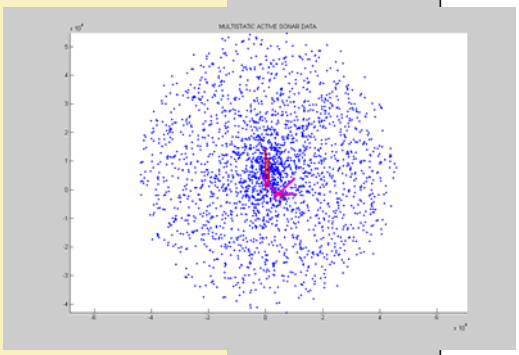
- Multi-sensor performance

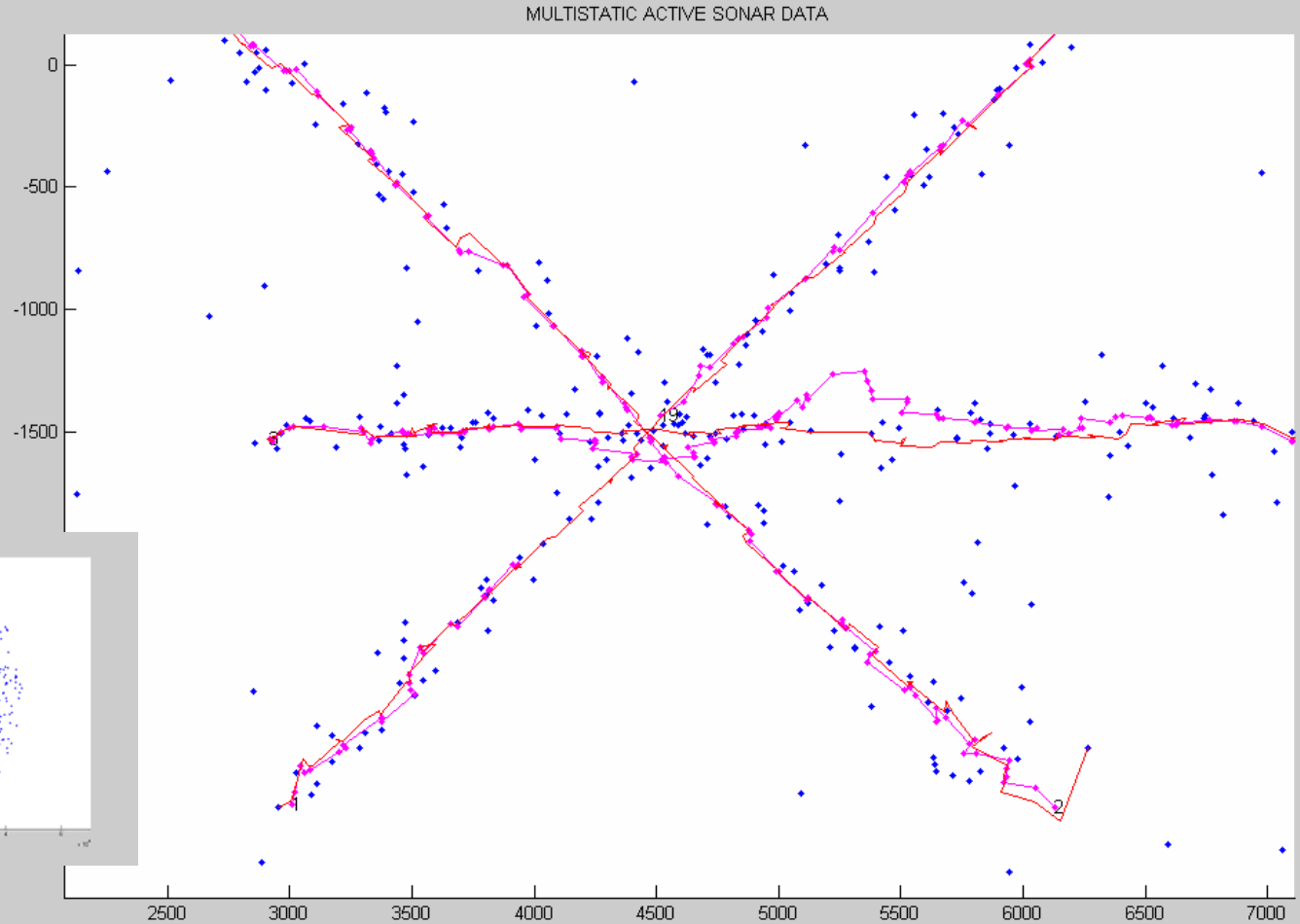




ROC curves

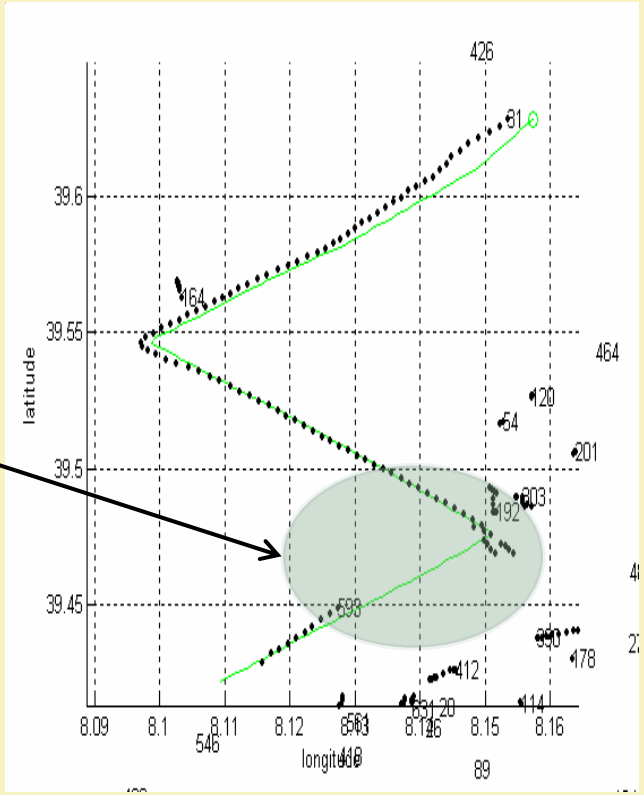
BASELINE VS. MHT





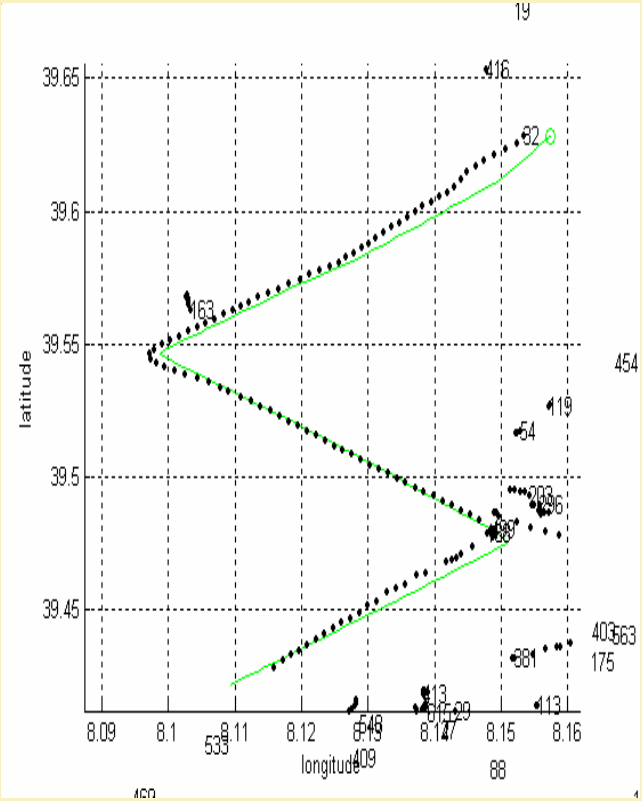
MANEUVERING TARGET IN CLUTTER

Without MHT processing



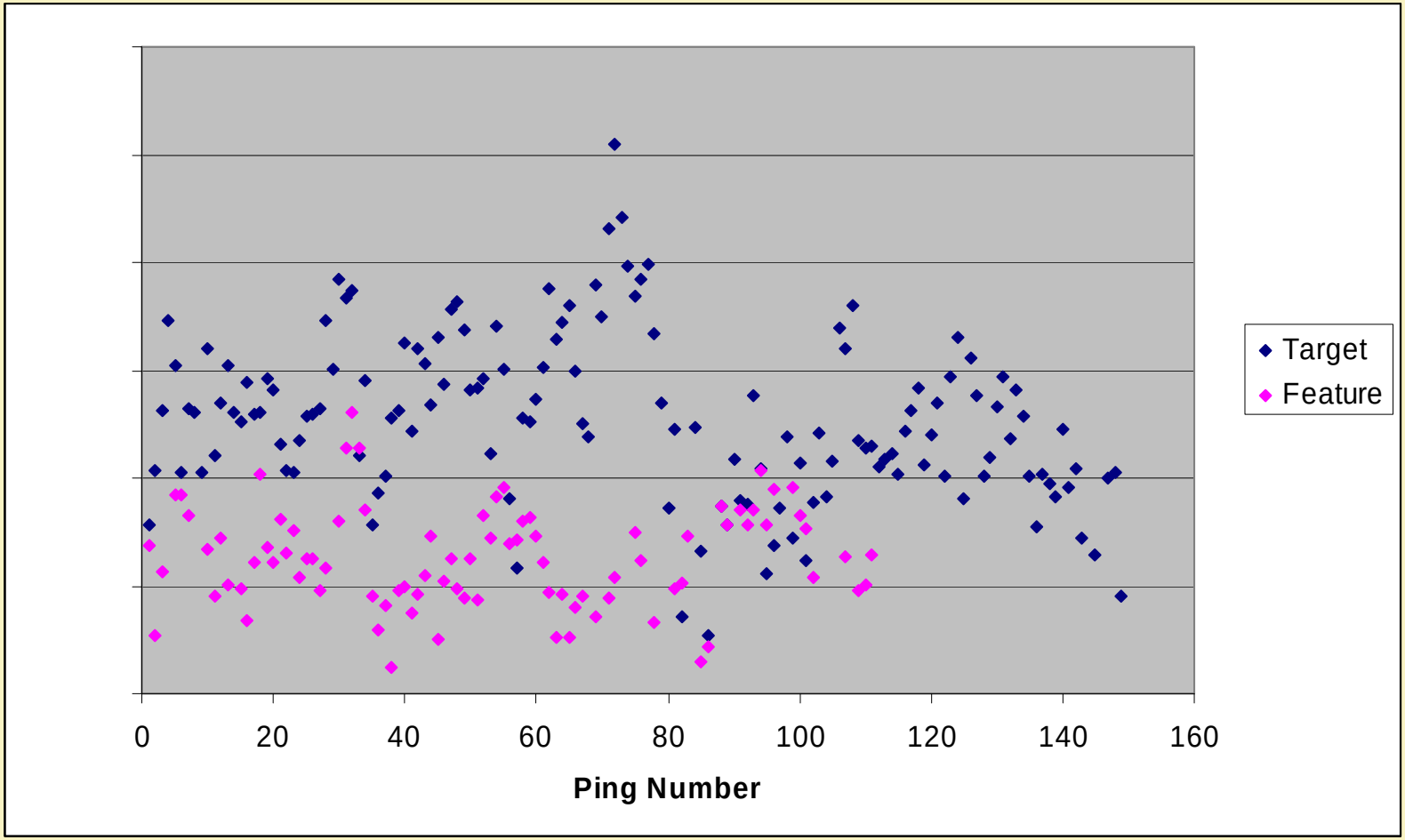
fragmentation

With MHT processing

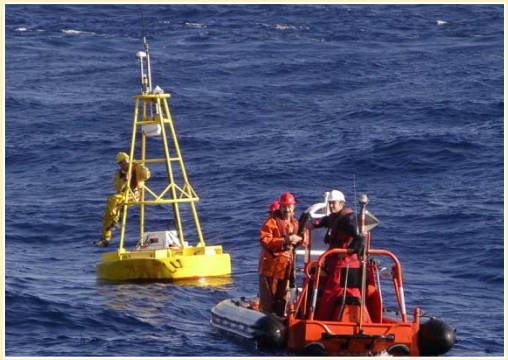


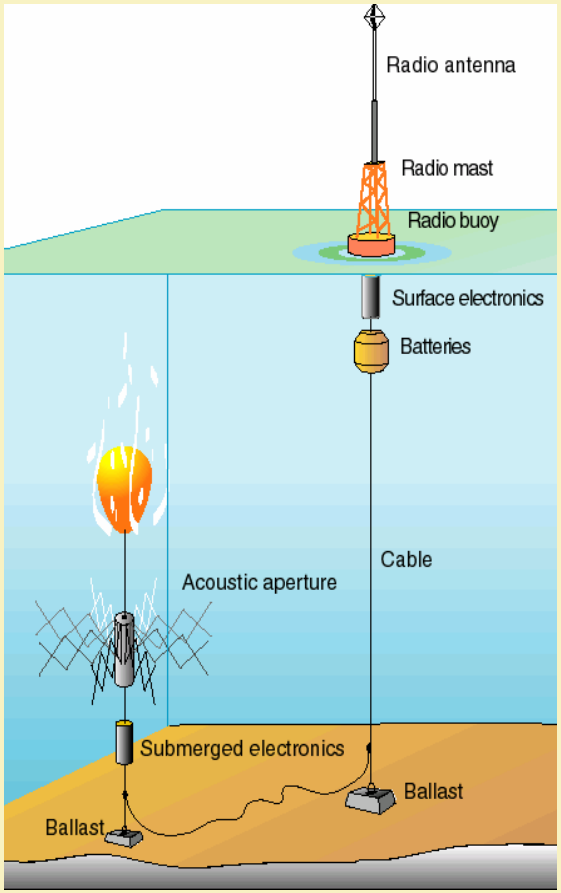
UNDERSEA SURVEILLANCE

FEATURE-AIDED TRACKING



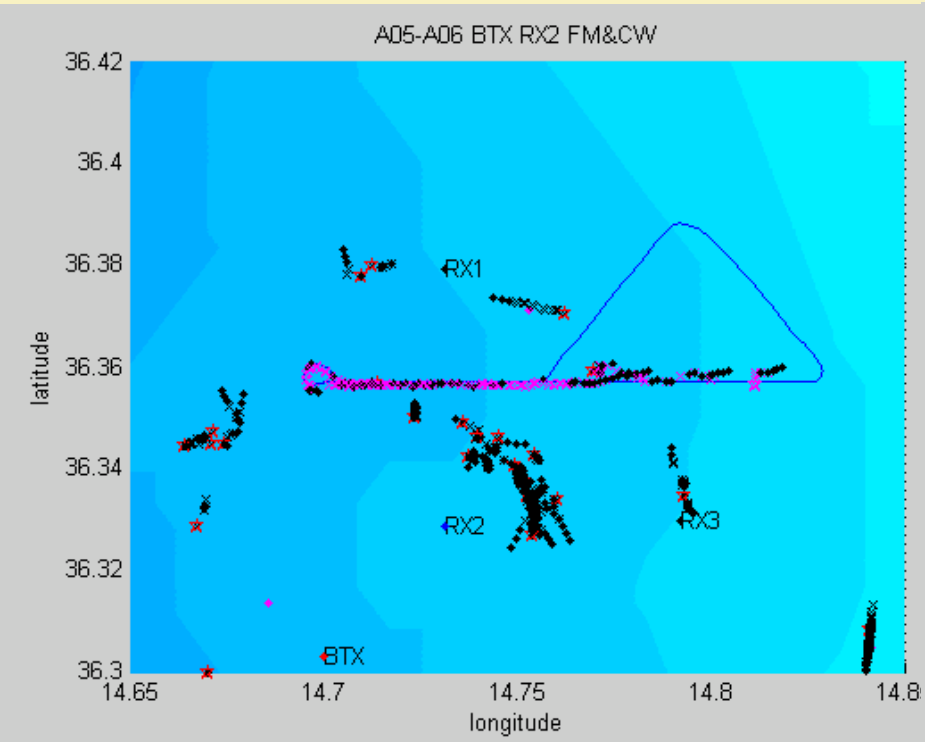
MULTI-WAVEFORM MULTI-SENSOR TRACKING

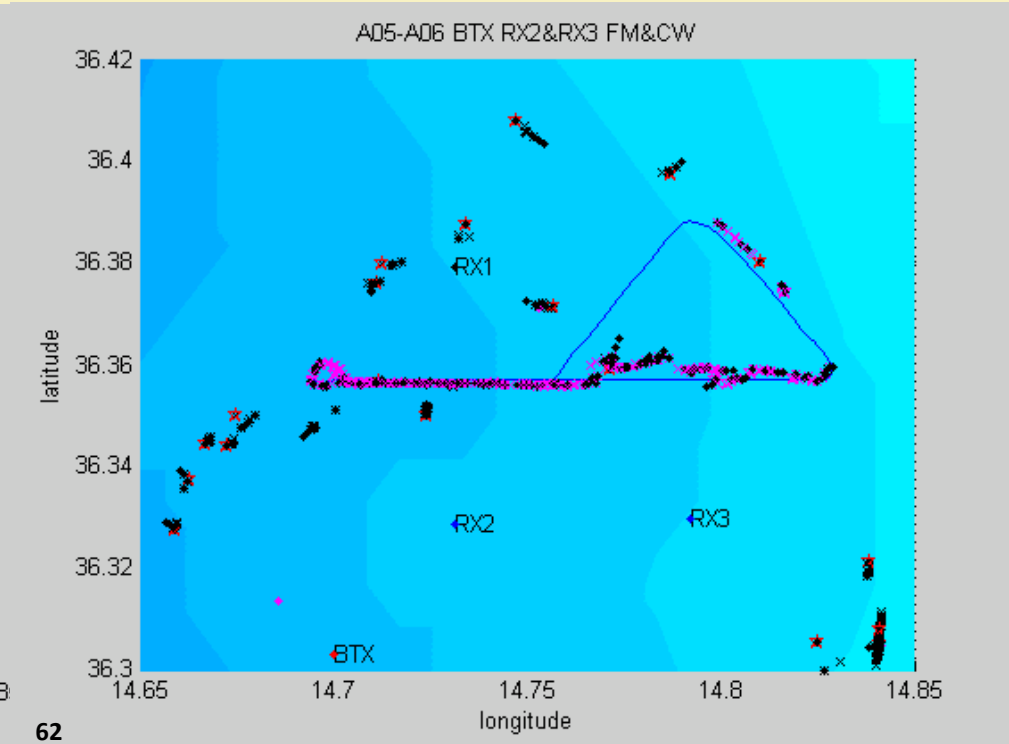




A CW-FM EXAMPLE

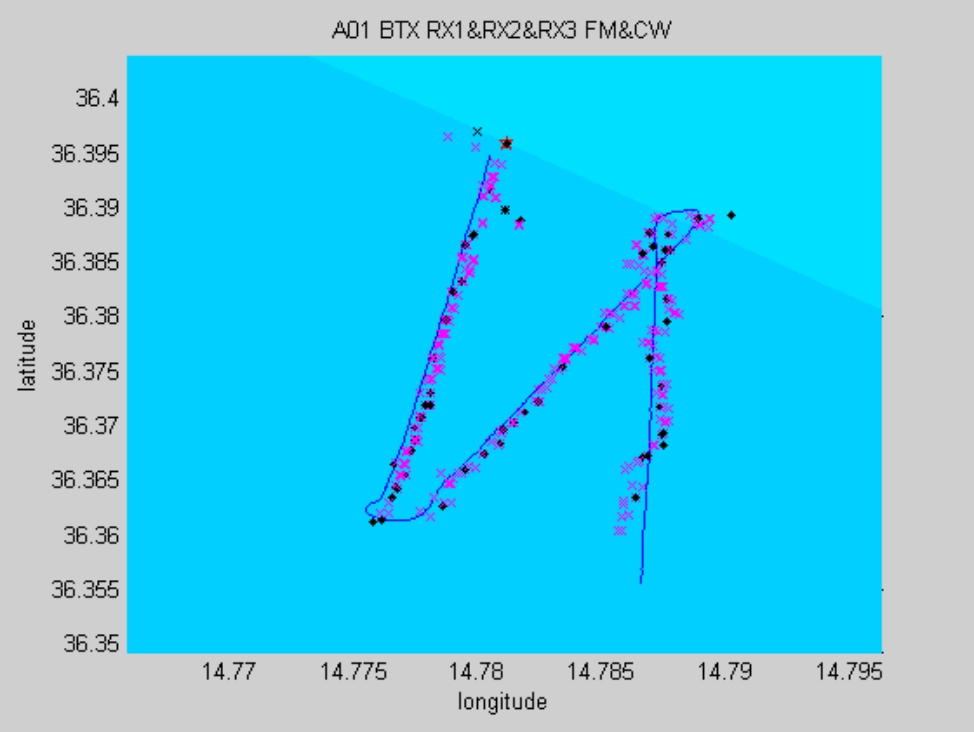
- Value-added of multiple source-receiver combinations
  - RX2 FM&CW: 72 tracks
  - RX2&RX3 FM&CW: 77 tracks (small increase), improved PD

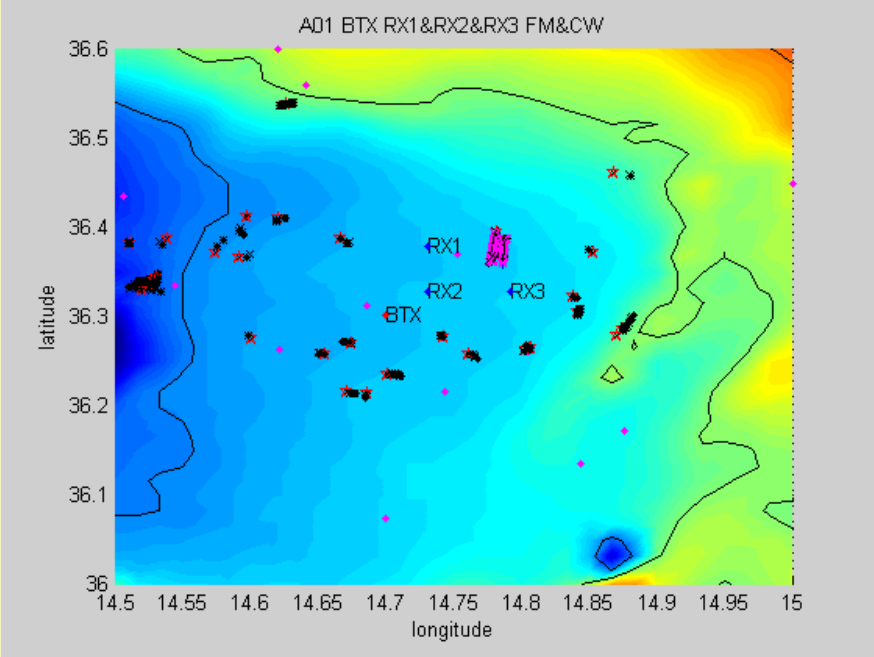




MULTI-WAVEFORM MULTI-SENSOR TRACKING

- ~83K contacts vs. 34 tracks
- Single well-localized target track

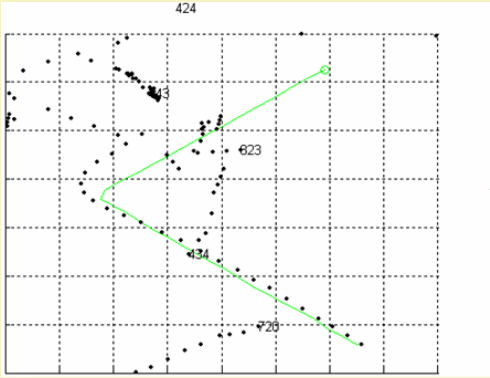




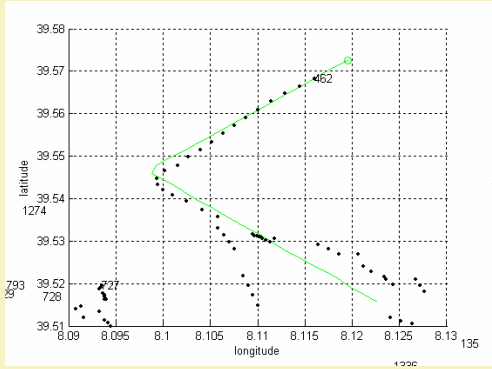
"Crazy Ivan" Run

CENTRALIZED FUSION (TRACK-WHILE-FUSE)

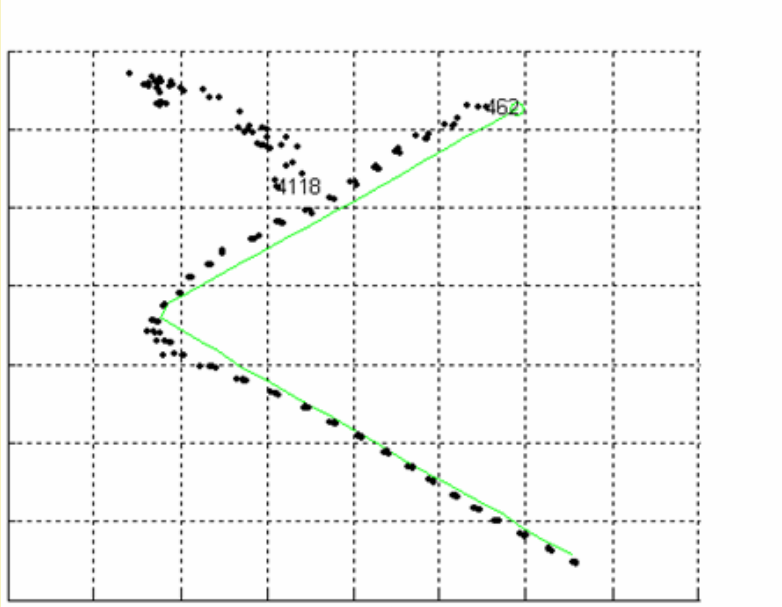
Very high FAR data
(approx. 200 per ping-source-receiver triple)



monostatic failure



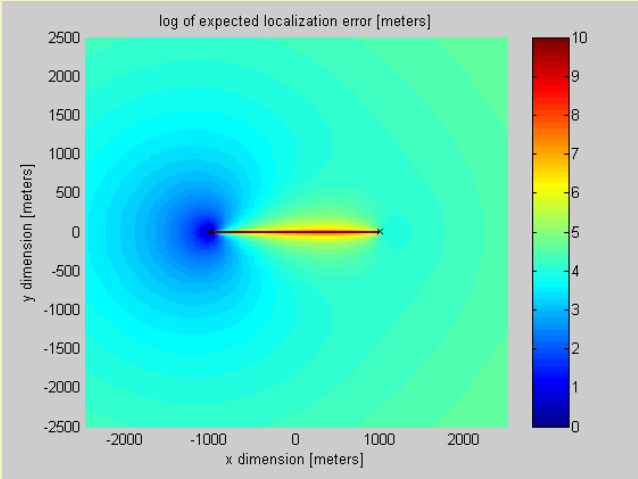
bistatic failure

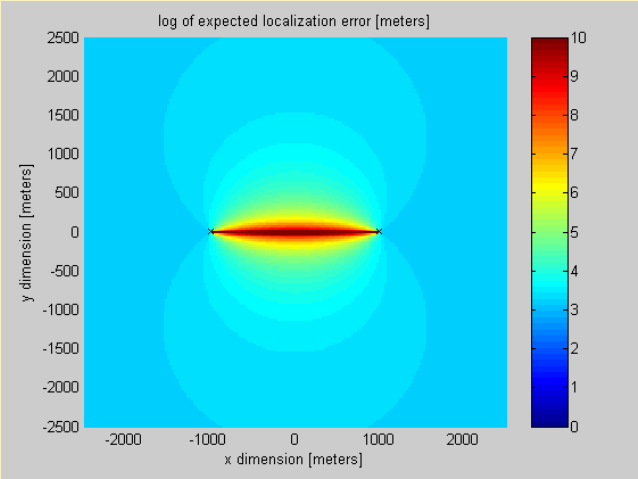


multistatic solution

INDIVIDUAL ERROR CONTRIBUTION

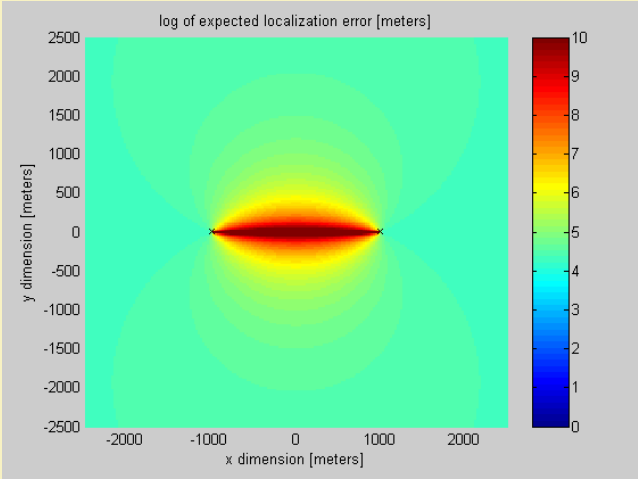
Bearing

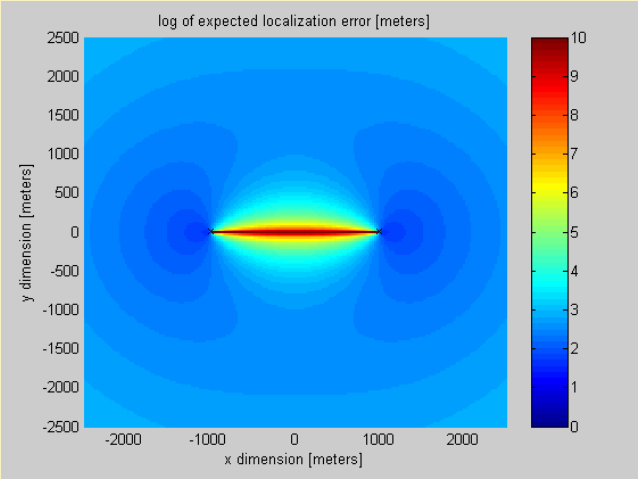




Source and receiver locations

Time





Speed of sound

MULTISTATIC LOCALIZATION

- Measurement model

$$Y = \begin{bmatrix} x^T & y^T \end{bmatrix}' \qquad X = \begin{bmatrix} x_{true}^T & y_{true}^T \end{bmatrix}'$$

$$Y = X + v \qquad v \sim N(0, \Sigma) \qquad \Sigma = \begin{bmatrix} \sigma_{x^T}^2 & \sigma_{x^T y^T} \\ \sigma_{x^T y^T} & \sigma_{y^T}^2 \end{bmatrix}$$

- Mean squared estimate – using $E\left[(Y_1 - X)(Y_2 - X)'\right] = 0$

$$\hat{X} = \Sigma_2 (\Sigma_1 + \Sigma_2)^{-1} Y_1 + \Sigma_1 (\Sigma_1 + \Sigma_2)^{-1} Y_2$$

$$\Sigma = \Sigma_1 (\Sigma_1 + \Sigma_2)^{-1} \Sigma_2$$

- Equivalent multistatic measurement model

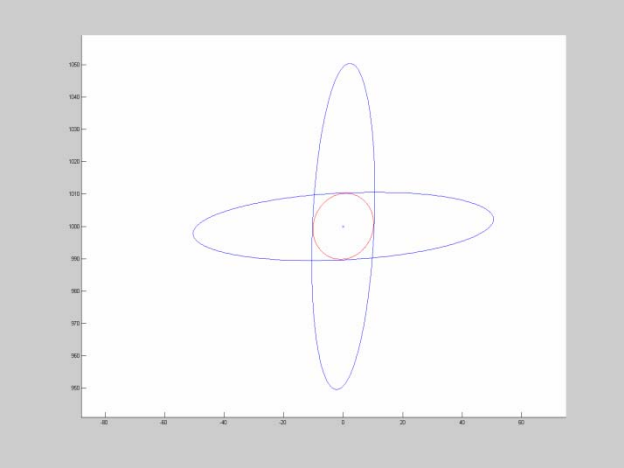
$$\hat{X} = X + v \qquad v \sim N\left(0, \Sigma_1 (\Sigma_1 + \Sigma_2)^{-1} \Sigma_2\right)$$

MULTISTATIC LOCALIZATION

$$Y = X + v \qquad v \sim N(0, \Sigma)$$

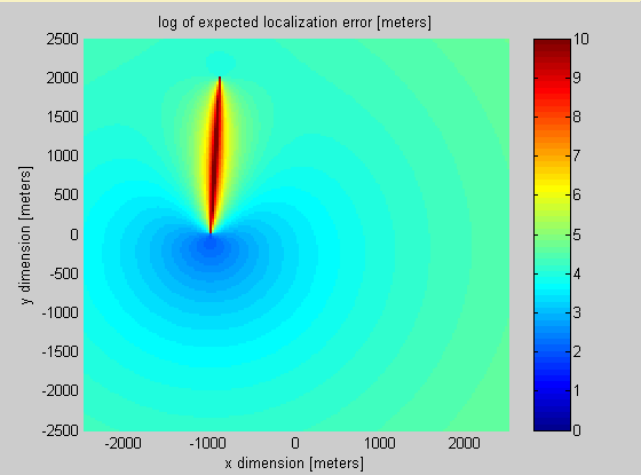
$$\hat{X} = X + v$$

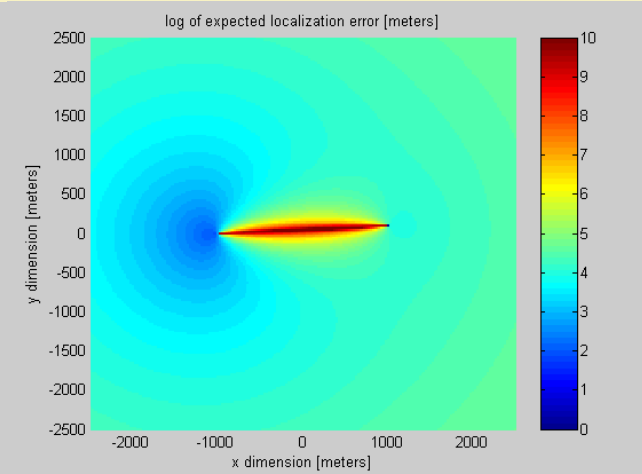
$$\begin{aligned}\hat{X} &= Y_1 + \Sigma_1 (\Sigma_1 + \Sigma_2)^{-1} (Y_2 - Y_1) \\ &= \Sigma_2 (\Sigma_1 + \Sigma_2)^{-1} Y_1 + \Sigma_1 (\Sigma_1 + \Sigma_2)^{-1} Y_2\end{aligned}$$

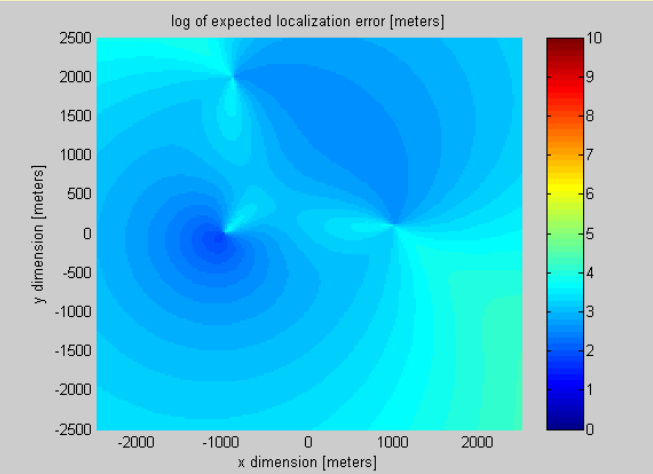


$$\sigma_\tau = 0.01\text{s}$$
$$\sigma_\theta = 1\text{deg}, \sigma_\varphi = 1\text{deg}$$

$$v \sim N\left(0, \Sigma_1 (\Sigma_1 + \Sigma_2)^{-1} \Sigma_2\right)$$







FUSED

OUTLINE

- Target modeling
- Bayesian filtering
- Multi-target filtering
- Multi-target tracking
- **Cardinality tracking**
- Distributed tracking
- Other topics

THE GREEDY-TARGET PROBLEM

- The problem: spurious tracks compete with good ones for sensor measurements
- A potential solution: favor confirmed or nearly-confirmed tracks in data association

tentative

nearly-confirmed

$$p(q^k \mid Z^k) = c_k \cdot p_\chi^\chi \left((1-p_\chi)(1-p_d)\right)^{\tau-\chi-d} \prod_{j\in J_t}\left[(1-p_\chi) f_d(z_j \mid Z^{k-1}, q^k)\right]$$

$$\cdot \prod_{j\in J_c}\left[(1-p_\chi)\xi_1 f_d(z_j \mid Z^{k-1}, q^k)\right] \cdot \prod_{j\in J_d}\left[(1-p_\chi)\xi_2 f_d(z_j \mid Z^{k-1}, q^k)\right]$$

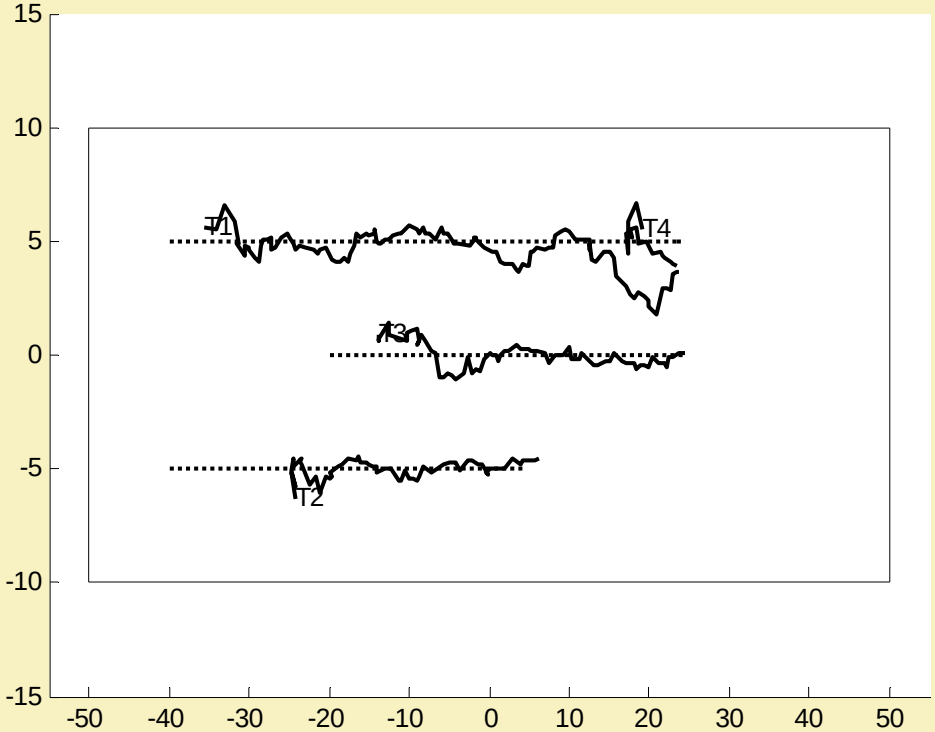
$$\cdot \prod_{j\in J_b}\left[\lambda_b f_b(z_j)\right] \cdot p(q^{k-1} \mid Z^{k-1}),$$

$$\xi_2 \geq \xi_1 \geq 1$$

confirmed

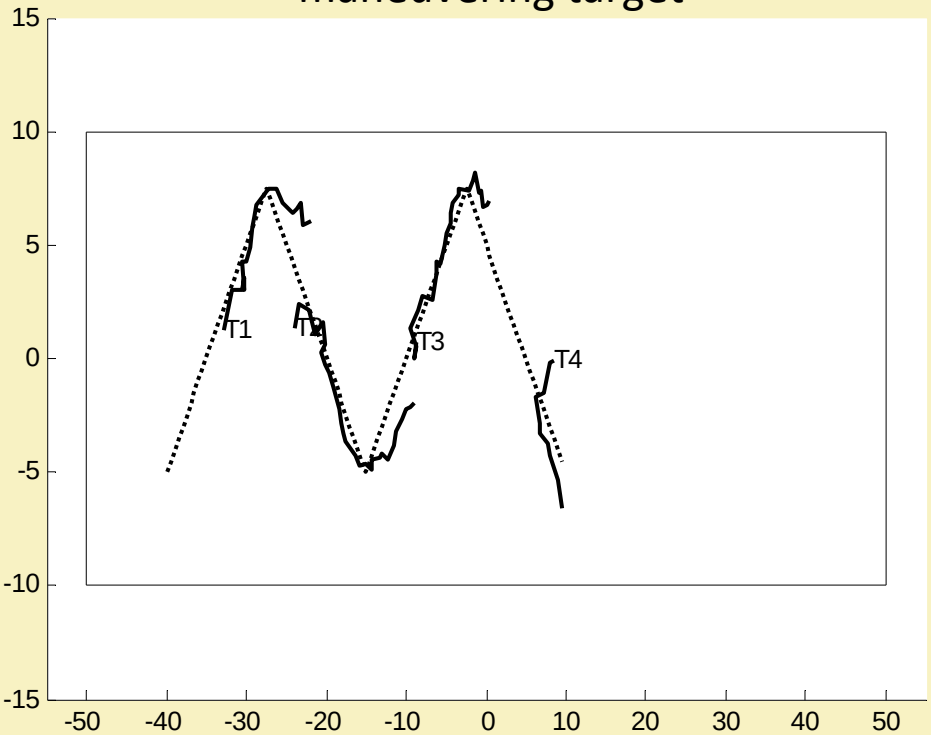
  - Note: we use $p_d \cdot \lambda_b$ as birth rate and assume all measurements used in data association stage
- Does it work?

SIMULATION RESULTS (1)

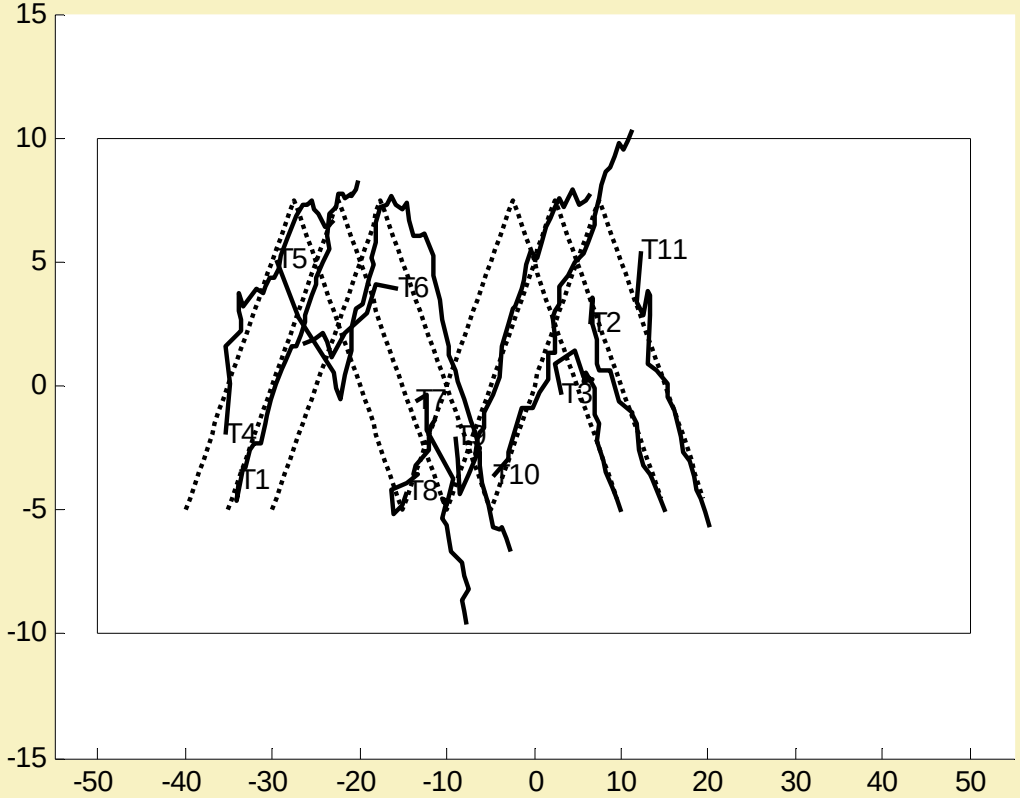


Scenario 1 – multiple non-maneuvering targets

Scenario 2 – single maneuvering target



SIMULATION RESULTS (2)



Scenario 3 – multiple maneuvering targets

Overall results (100 realizations of each scenario)

Metric	Track PD	Track Quality	Track Purity	Track Rate	Track Loc. Error
Percent change	2.43%	5.34%	8.10%	-6.09%	-3.02%

WHAT IS THE JUSTIFICATION FOR MODIFIED MHT?

- Limiting form of tracking problem: cardinality tracking

$$p\left(q^k \mid Z^k\right) = \frac{p_\chi^\chi \left(1-p_\chi\right)^{\tau-\chi} \left(1-p_d\right)^{\tau-\chi-d} p_d^{b+d} \lambda_b^b}{\lambda_{fa}^{d+b}} \frac{p\left(q^{k-1} \mid Z^{k-1}\right)}{\bar{c}_k}$$

- If $p_d = 1$
  - Optimality is achieved by batch extraction of tracks out of a Tetris structure with length of at least k_0

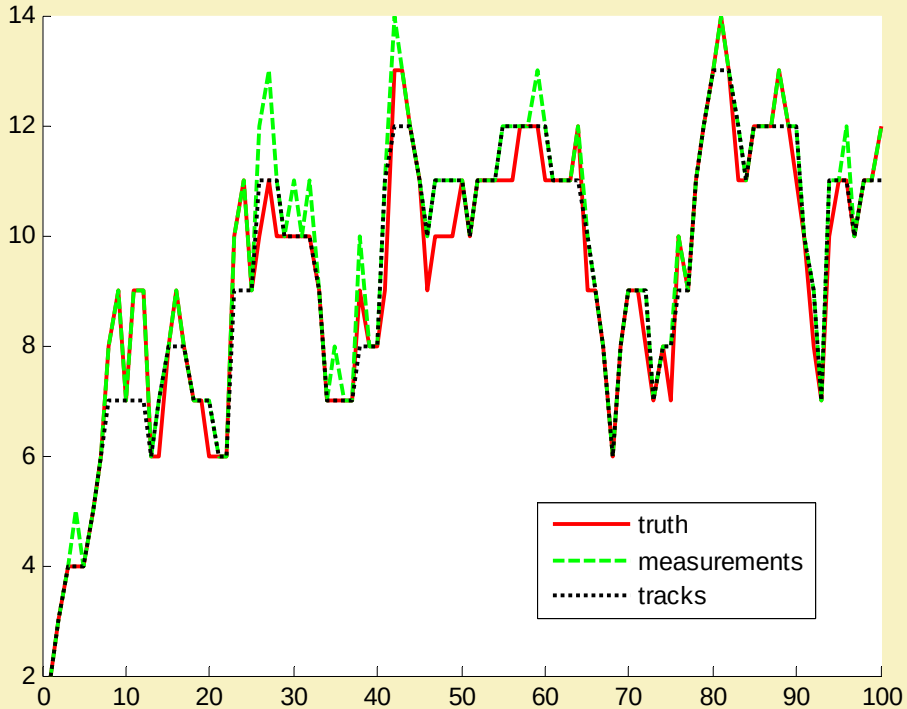
$$k_0 = \min_i \left\{ i, \frac{\lambda_b \left(1-p_\chi\right)^{i-1} p_\chi}{\lambda_{fa}^i} \geq 1 \right\}$$

		1																	
		1																	
		1	1																
		1	1	1										1				1	
	1	1	1	1		1	1	1				1	1	1		1		1	
1	1	1	1	1	1	1	1	1	1			1	1	1	1	1		1	1

The Tetris structure

  - In sequential processing using the modified MHT equations, optimality is achieved provided $n - scan \geq k_0 - 3$

A CARDINALITY TRACKING EXAMPLE



Parameter description	Setting
Target birth rate	1
Target death probability	0.1
Sensor probability of detection	1
Sensor false alarm rate	0.33
Track initiation	3-of-3
Hypothesis tree depth (*n-scan*)	0
Confirmation reward factor (modified MHT)	2, 1.5
Track termination (maximum missed detections)	0
Number of scans in each scenario realization	100
Number of realizations	1000

$$\frac{\lambda_b (1-p_\chi)^2 p_\chi}{\lambda_{fa}^3} > 1 > \frac{\lambda_b (1-p_\chi) p_\chi}{\lambda_{fa}^2} \Rightarrow k_0 = 3$$

$$k_0 = 3 \Rightarrow n-scan = 0$$

- For optimal track extraction with arbitrary p_d and for large-scale surveillance, cardinality tracking requires an equivalence-class approach

LARGE-SCALE CARDINALITY TRACKING

- The equivalence-class approach: we seek the likeliest set of equivalent hypotheses

$$p(\{q^k\} \mid Z^k) = \frac{p(Z_k \mid Z^{k-1}, \{q^k\}) p(\{q^k\} \mid Z^{k-1})}{c_k} = \frac{p(Z_k \mid Z^{k-1}, \{q^k\}) p(\{q_k\} \mid Z^{k-1}, \{q^{k-1}\}) p(\{q^{k-1}\} \mid Z^{k-1})}{c_k}$$

$$c_k = p(Z_k \mid Z^{k-1}) = \sum_{\{q^k\}} p(Z_k \mid Z^{k-1}, \{q^k\}) p(\{q^k\} \mid Z^{k-1})$$

- The result – the key change from before is in the multiplicative weights

$$p(\{q^k\} \mid Z^k) = \frac{p_\chi^\chi (1-p_\chi)^{\tau-\chi} (1-p_d)^{\tau-\chi-d} p_d^{b+d} \lambda_b^b}{\lambda_{fa}^{d+b}} \frac{p(\{q^{k-1}\} \mid Z^{k-1})}{\bar{c}_k} \cdot \left(\prod_{i=1}^{d} (r+1-i) \right) \left(\prod_{i=1}^{b} (r-d+1-i) \right)$$

- Optimal track extraction from a provably optimal Tetris configuration is straightforward

FURTHER APPROACHES

- More aggregation
  - "Psi tracking" – we don't care about which tracks are updated
  - "Aggregate psi tracking" – we care only about the difference between births and deaths
  - No structural results nor optimal extraction are possible with these further levels of hypothesis aggregation
- The Kalman filter

$$x(1|0) = \frac{\lambda_b}{p_\chi} \qquad \Sigma(1|0) = \frac{\lambda_b}{p_\chi}$$

$$x(k+1|k) = (1-p_\chi)x(k|k) + \lambda_b \qquad \Sigma(k+1|k) = (1-p_\chi)^2\Sigma(k|k) + \lambda_b$$

$$L(k+1) = \frac{p_d\Sigma(k+1|k)}{p_d\Sigma(k+1|k) + \lambda_{fa}}$$

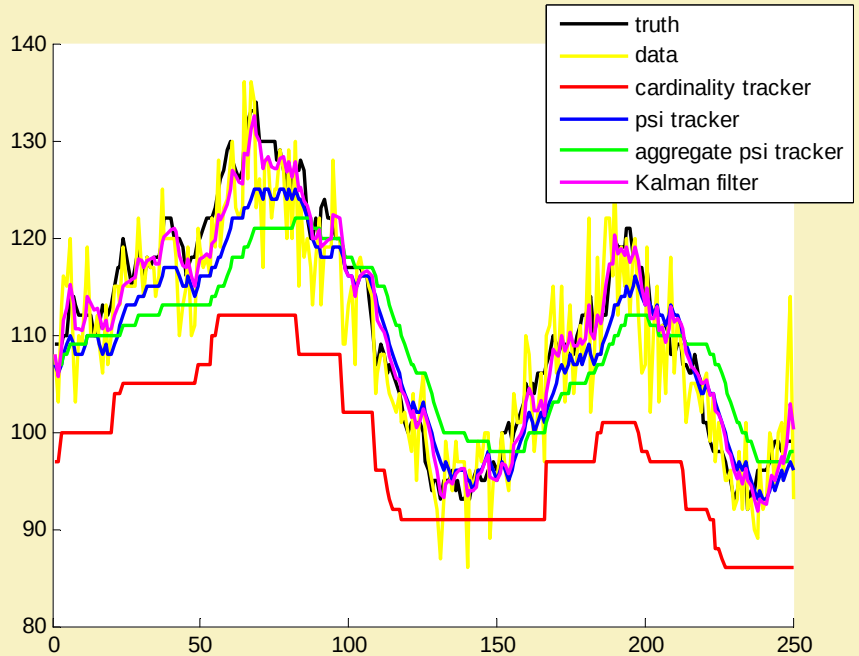
$$x(k+1|k+1) = x(k+1|k) + L(k+1)\left(Z_{k+1} - p_d X(k+1|k) - \lambda_{fa}\right) \qquad \Sigma(k+1|k+1) = \left(I - L(k+1)p_d\right)\Sigma(k+1|k)$$

NUMERICAL RESULTS

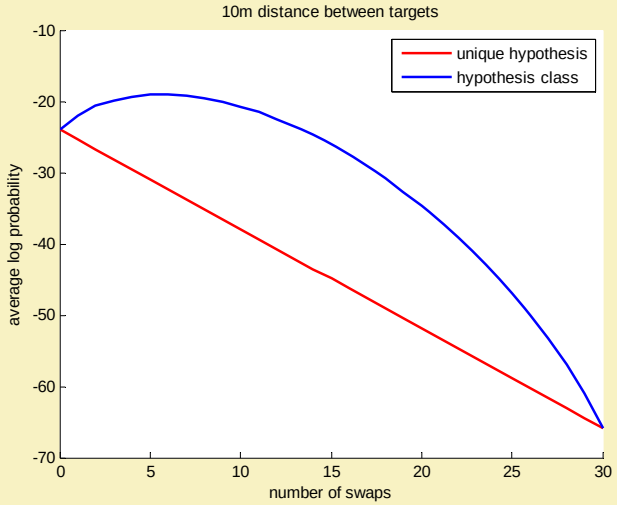
Monte of Monte Carlo iterations	10
Number of scans of data per realization	$N = 250$
Probability of death	$p_\chi = 0.01$
Birth rate	$\lambda_b = 1$
False alarm rate	$\lambda_{fa} = 10$

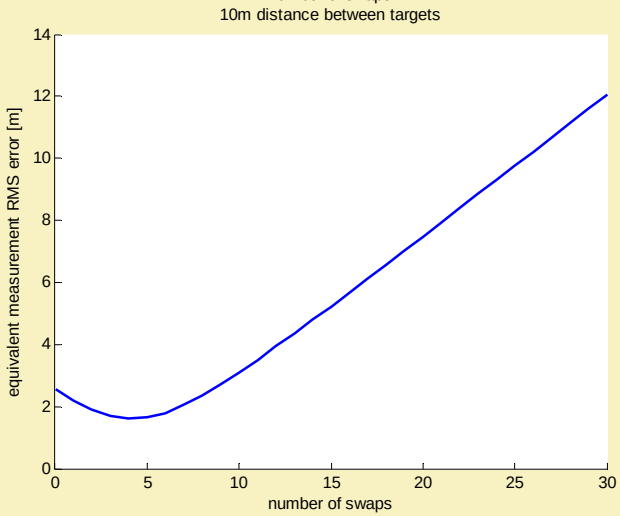
$$E[N_T] = \frac{\lambda_b}{p_\chi} + (N-1)\lambda_b$$

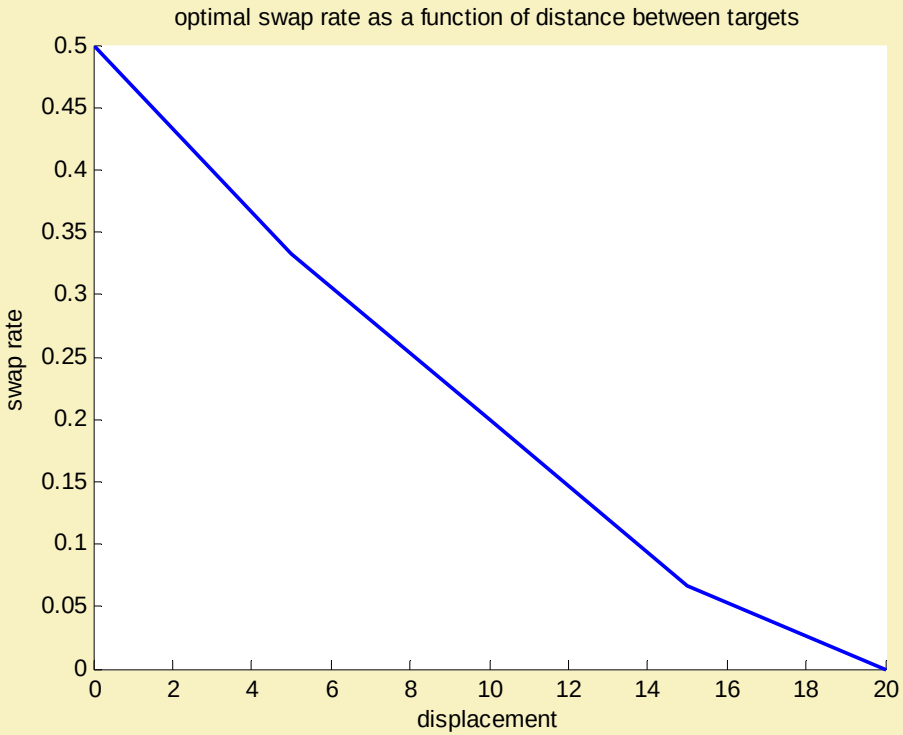
estimation scheme	track cardinality error
a priori estimate	9.9716
cardinality tracker	11.1536
psi tracker	2.5020
aggregate psi tracker	3.3508
Kalman filter	2.0072



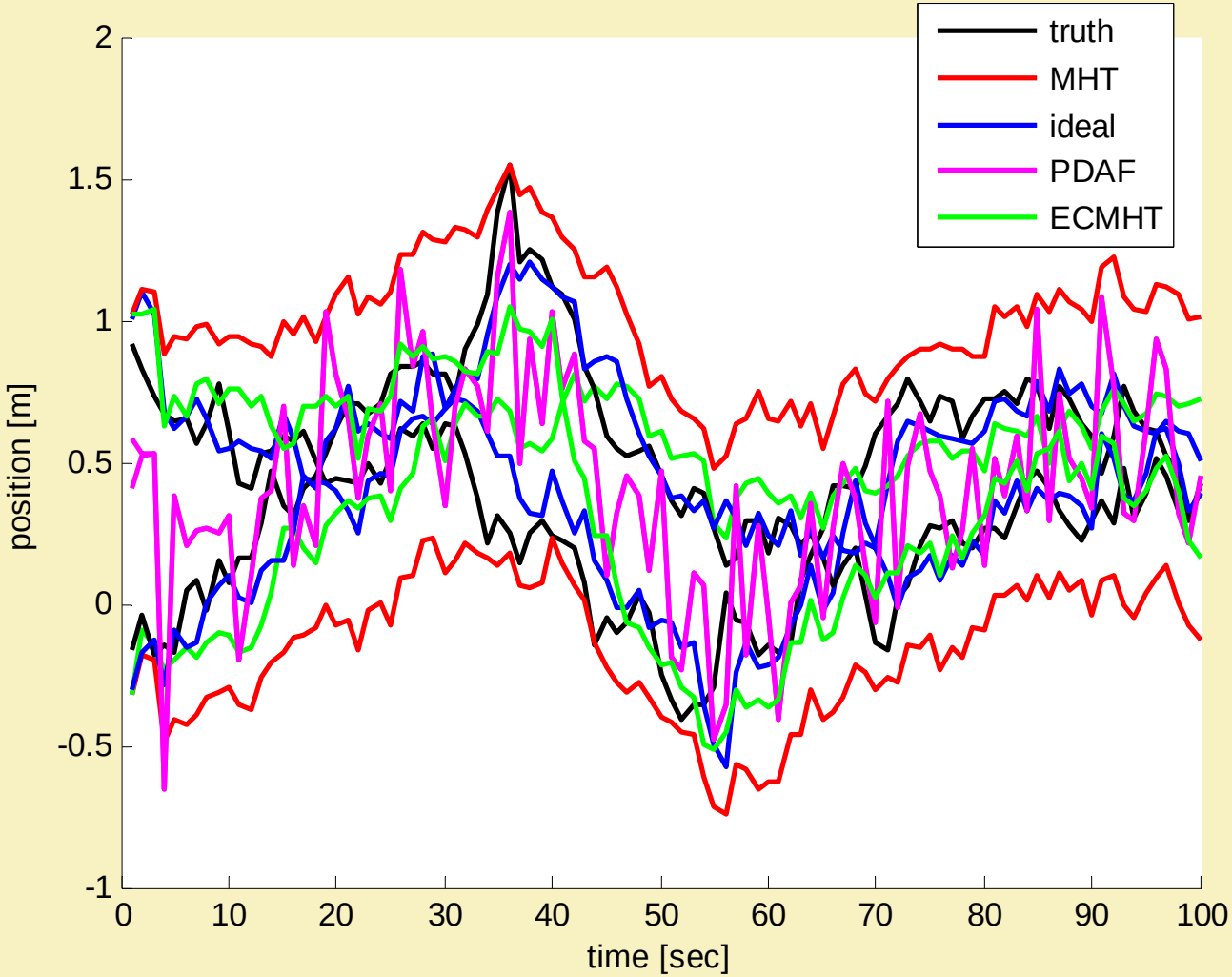
EQUIVALENCE-CLASS MHT







AN ILLUSTRATION



CARDINALITY TRACKING: A SPECIAL CASE

- Assumptions (no false alarms, one scan)
  - Targets $N_T \sim Poisson(\lambda_T)$ $p(N_T = n_T) = \frac{\lambda_T^{n_T}}{n_T!}\exp(-\lambda_T)$ $n_T \geq 0$
  - Detections $N_C \sim Poisson(p\lambda_T)$ $p(N_C = n_C) = \frac{(p\lambda_T)^{n_T}}{n_T!}\exp(-p\lambda_T)$ $n_C \geq 0$
- Given that n_C measurements are observed, how many targets are there?
  - Limiting case of tracking problem: no measurement information save for the cardinality of the measurement set

THE SOLUTION

- Bayesian decomposition

$$p(N_T = n_T \mid N_C = n_C) = \frac{p(N_C = n_C \mid N_T = n_T)\, p(N_T = n_T)}{p(N_C = n_C)} \qquad n_T \geq 0 \qquad n_C \geq 0$$

$$p(N_C = n_C \mid N_T = n_T) = \binom{n_T}{n_C} p^{n_C} (1-p)^{n_T - n_C} \qquad n_T \geq n_C$$

$$p(N_T = n_T \mid N_C = n_C) = \frac{((1-p)\lambda_T)^{n_T - n_C} \exp(-(1-p)\lambda_T)}{(n_T - n_C)!} \qquad n_T \geq n_C \geq 0$$

- Result

$$N_T = n_C + r \qquad r \sim Poisson((1-p)\lambda_T)$$

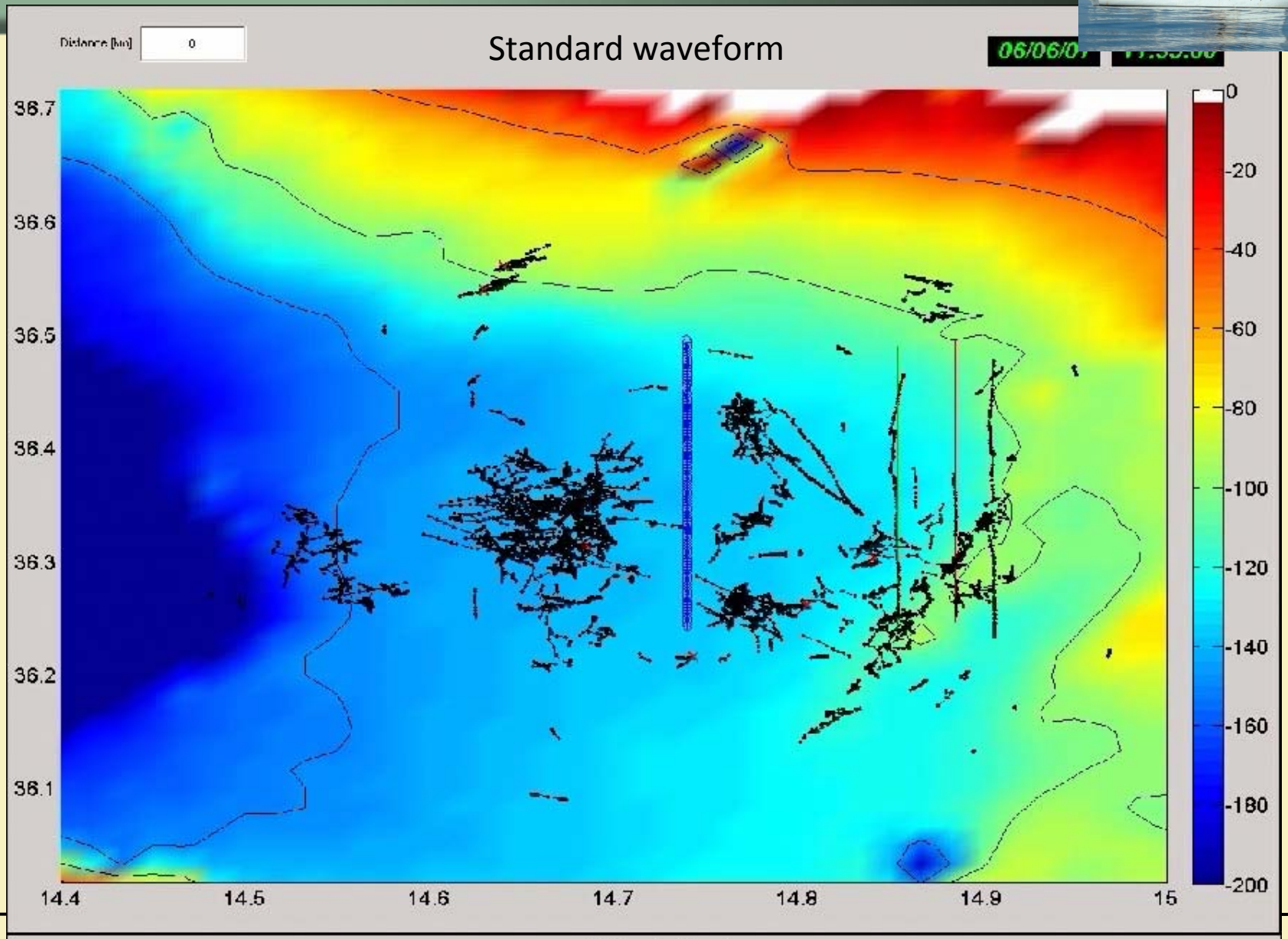
- Notes
  - *Residual* number of targets is Poisson distributed
  - Poisson prior does not reflect interdependencies that may exist regarding object presence (e.g. mine warfare literature)

OUTLINE

- Target modeling
- Bayesian filtering
- Multi-target filtering
- Multi-target tracking
- Cardinality tracking
- **Distributed tracking**
- Other topics

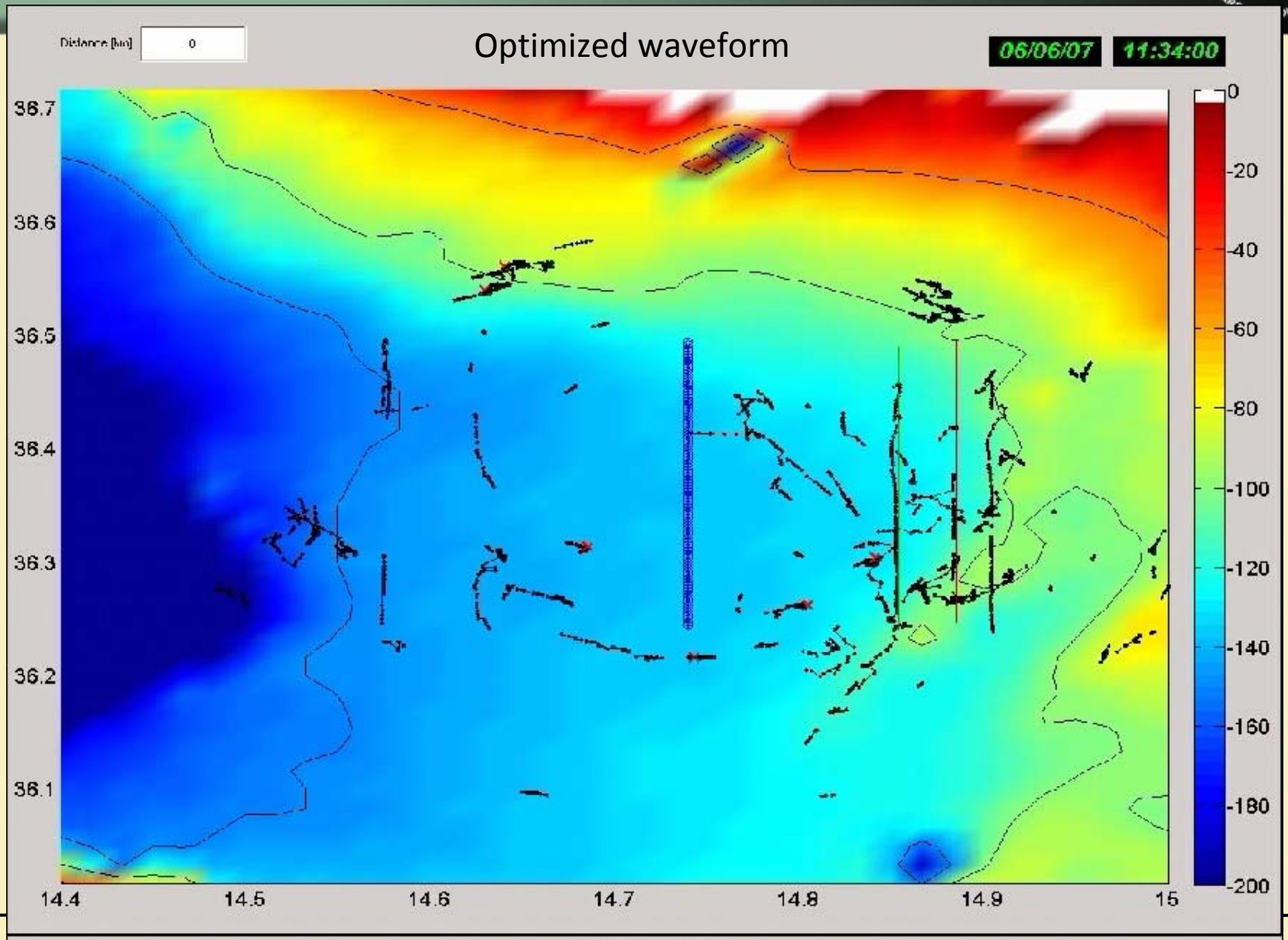
compunetix

Standard waveform



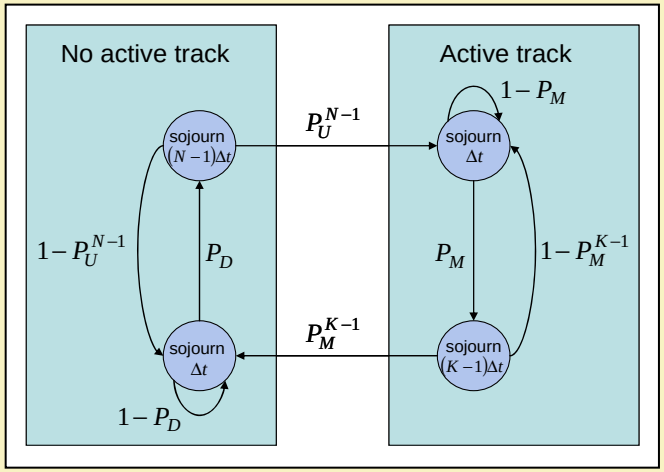
runB10_Standard_2.avi

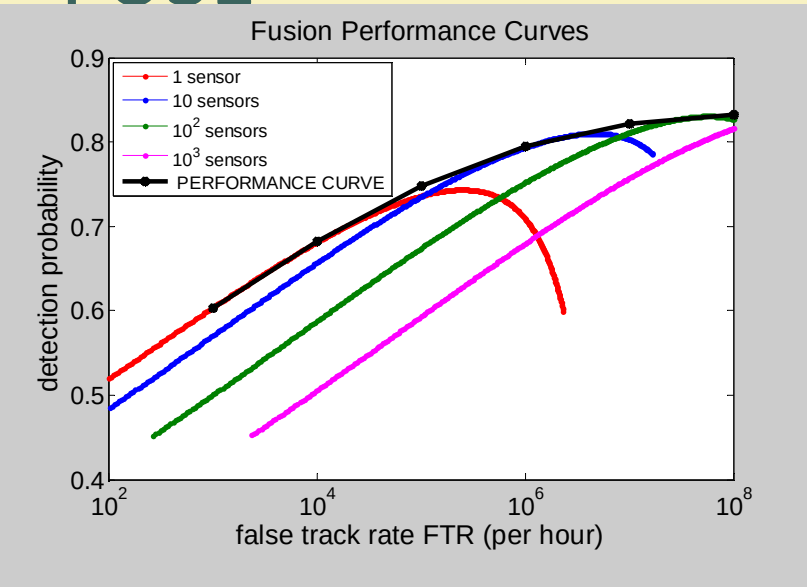
Optimized waveform

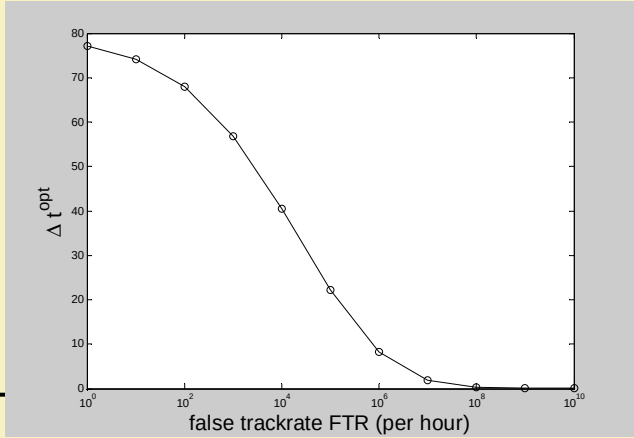


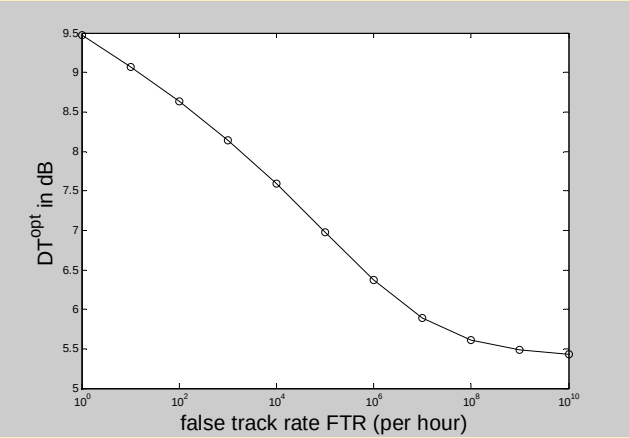
runB10_Adapted_1.avi

PERFORMANCE LIMITS IN TRACK-WHILE-FUSE



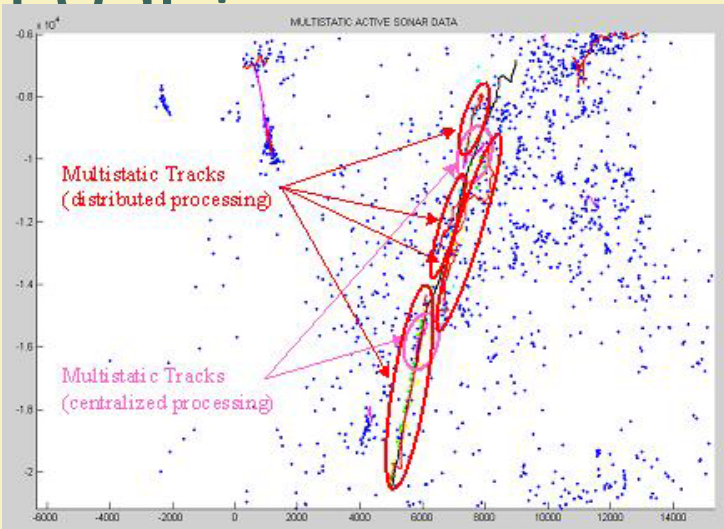




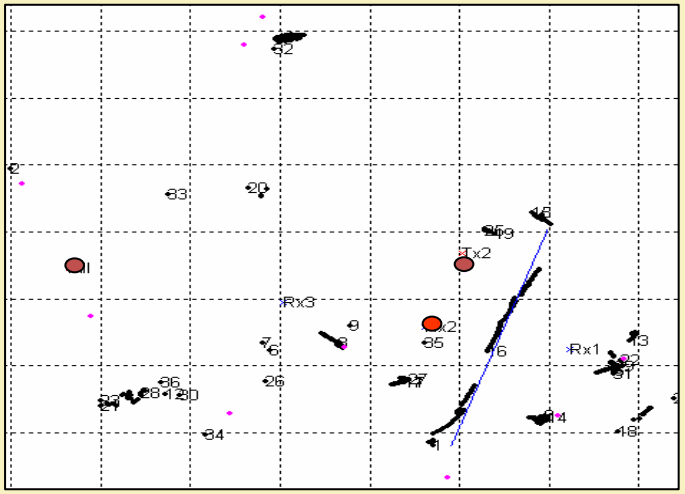


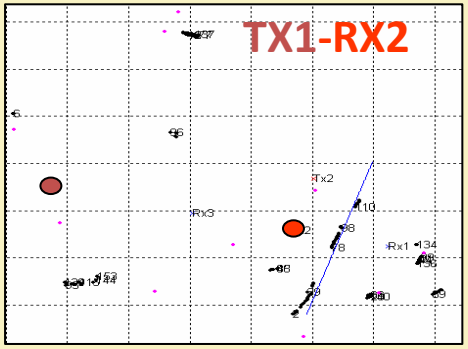
SO WHAT CAN GO WRONG IN TRACK-WHILE-FUSE?

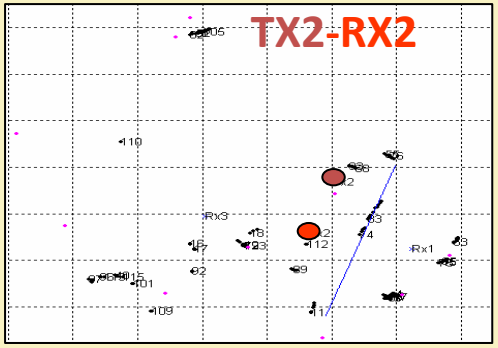
- Registration errors



- Target fading effects





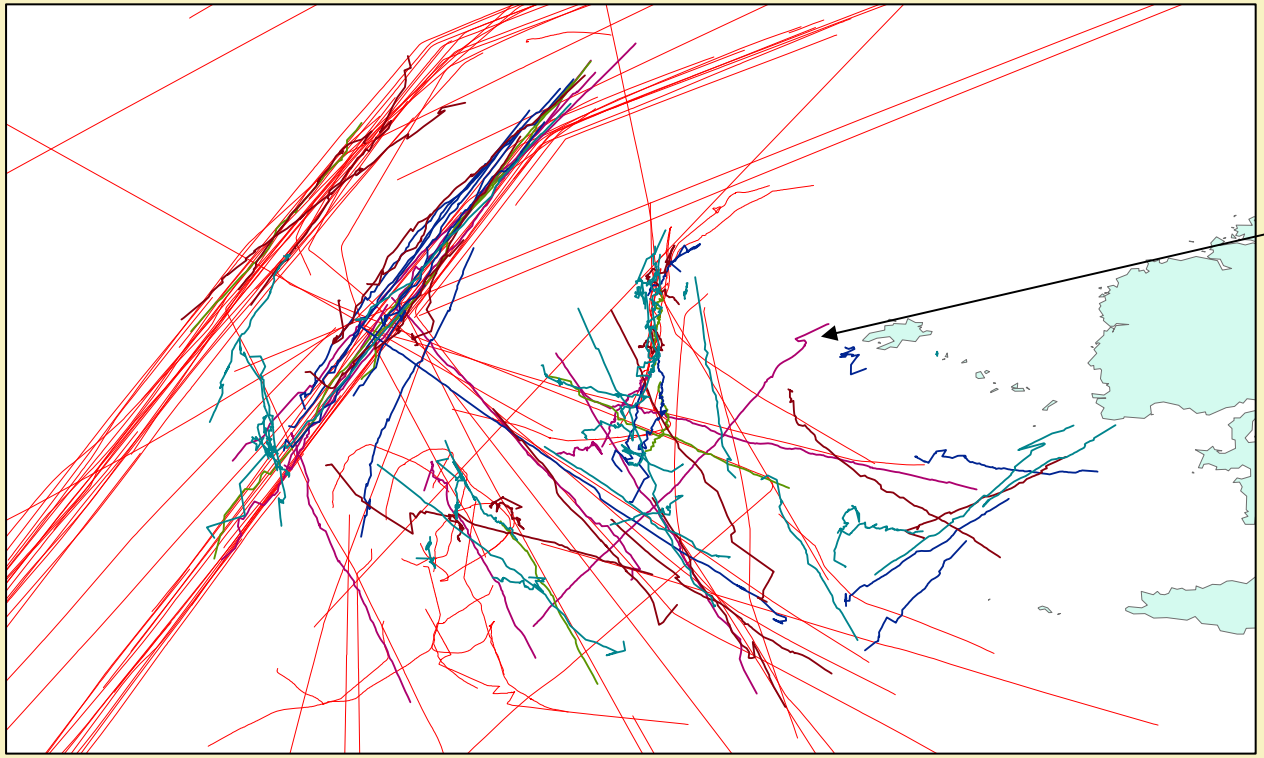


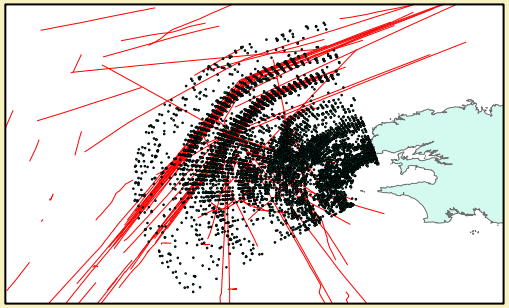
CENTRALIZED VS. DISTRIBUTED FUSION

- Optimal trackers don't exist
- Near-optimal MHT solutions can be developed but are often difficult to design and implement
  - *Challenging conditions* (high clutter, dim targets, poor sensor resolution): adaptive MHT solutions may be difficult to develop
  - *Persistent multi-target ambiguities*: single-window (fixed or variable) MHT processing not always best way to prune the hypothesis tree
- Advanced MHT processing can be achieved in a simple way be concatenating modular MHT modules – thus *track fusion* is a scan-based, contact-based, data-association function

ADAPTIVE MHT VIA MULTI-STAGE PROCESSING

- Targets in heavy clutter: *track-extract-track*



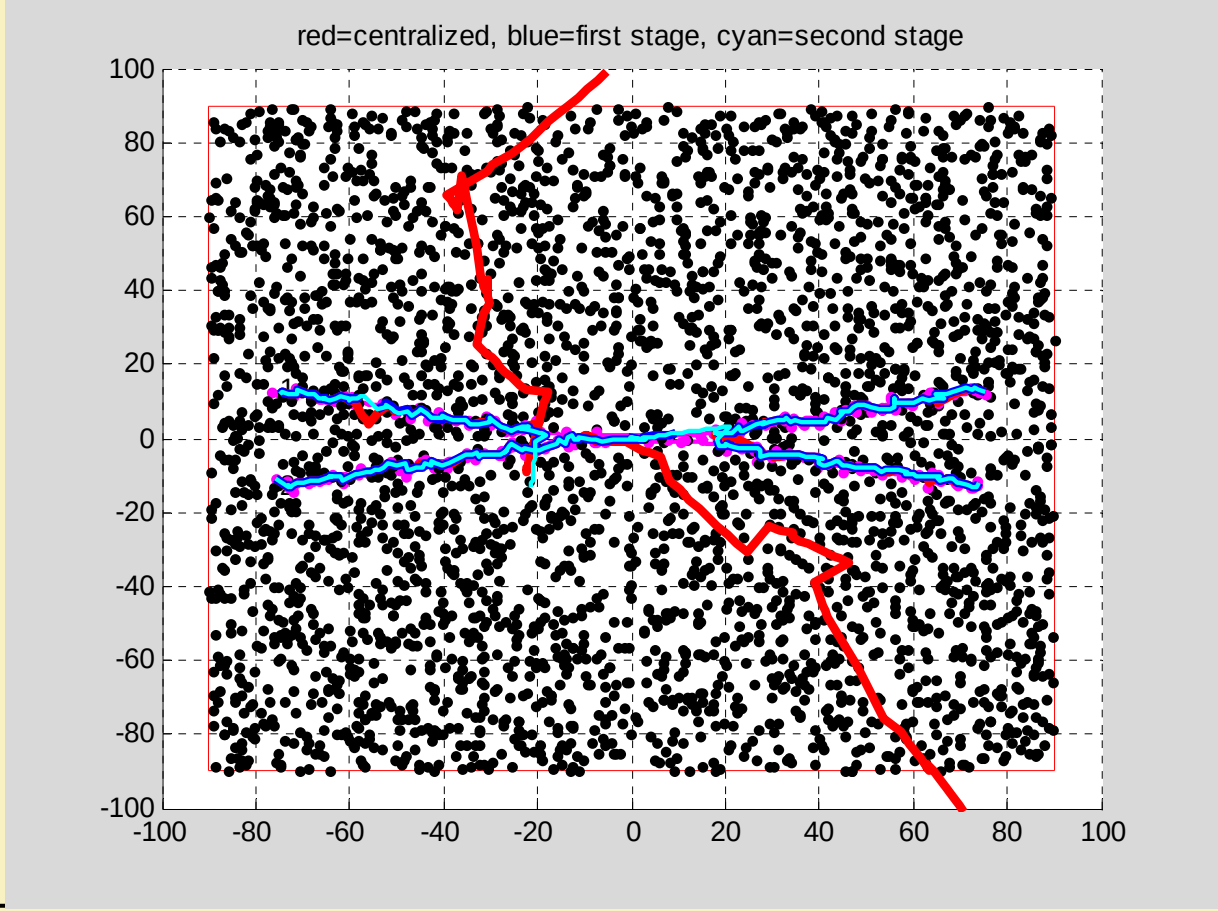


Large unidentified ship track

DIM TARGETS

TRACK-ADAPT-TRACK

- First stage: near-zero process noise
- Second stage: larger process noise, longer coasting times, and larger hypothesis tree depth



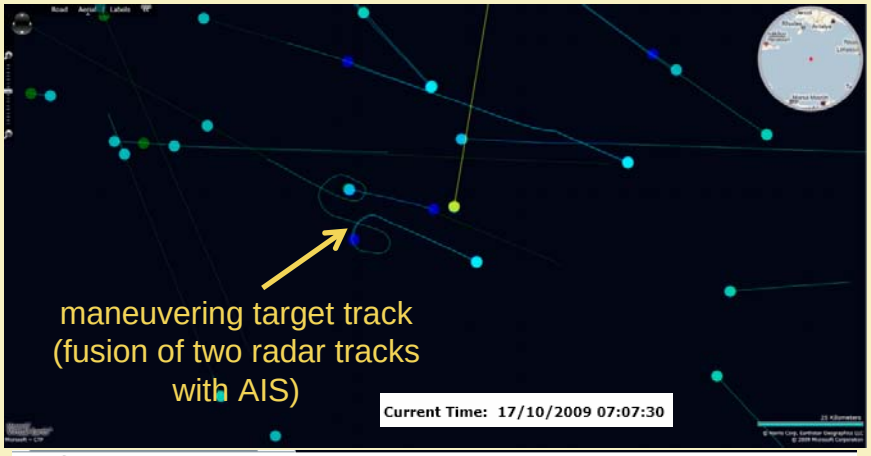
PERSISTENT MULTI-TARGET AMBIGUITIES

- **Multi-scale data: *track-before-fuse* & *track-break-fuse***
- Slowly crossing targets: *track-break-fuse*
- Large sensor networks: *fuse-before-track*
- Group tracking: *track-segment-fuse*

SURVEILLANCE WITH MULTI-SCALE DATA AND LEGACY DATA

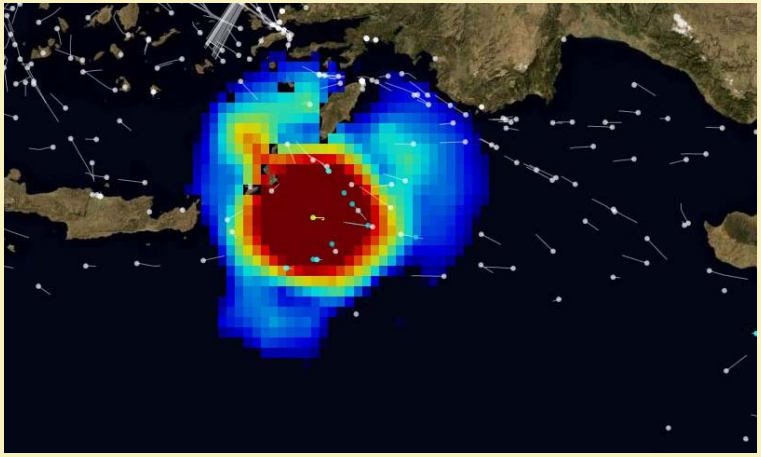
The solution: *track-before-fuse*

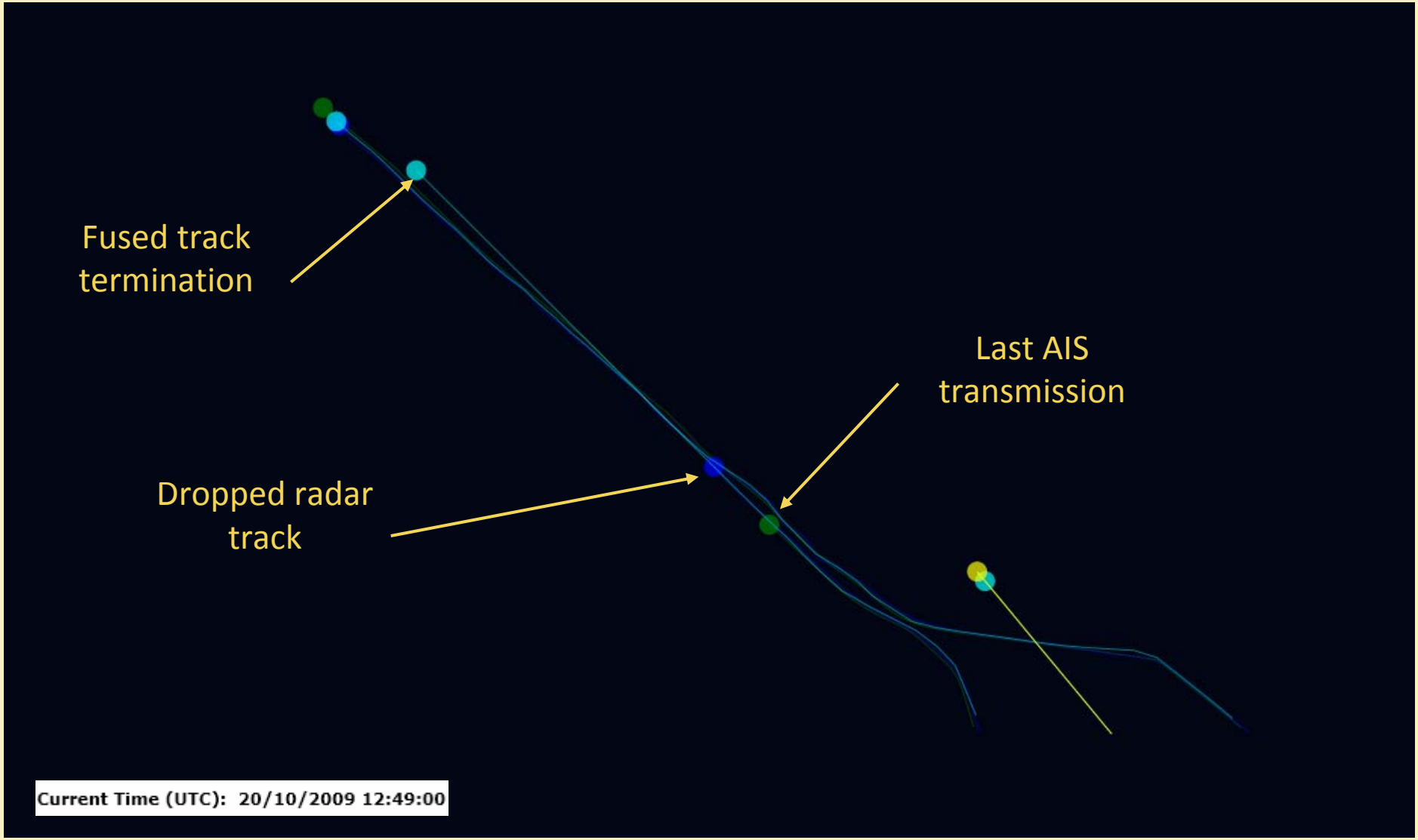
- Seamless wide-area data processing
- Cooperative (AIS, SIGINT) and non-cooperative (radar, SAR imagery, sonar) sources
- Modular (multi-stage) multi-hypothesis tracking

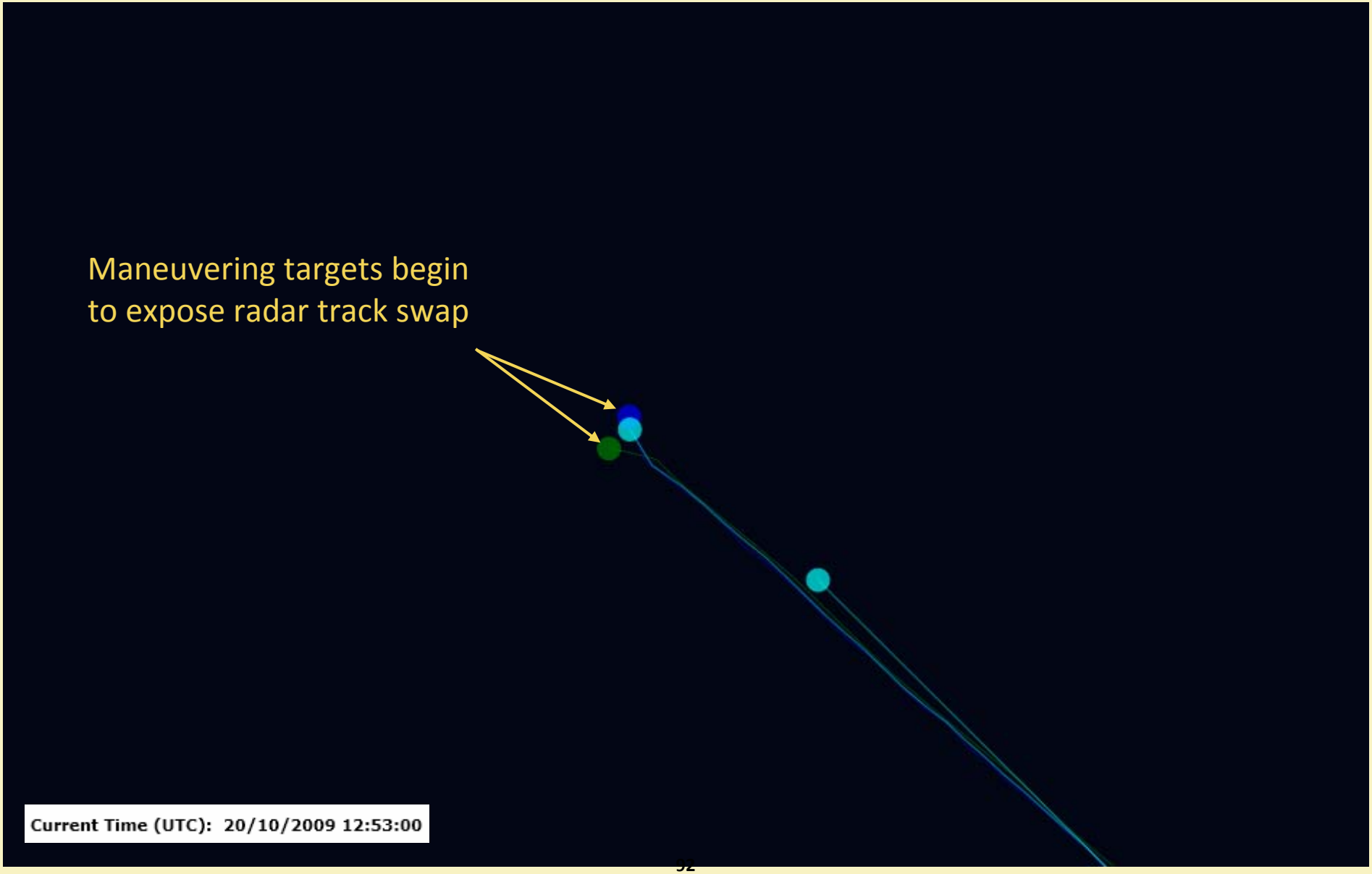


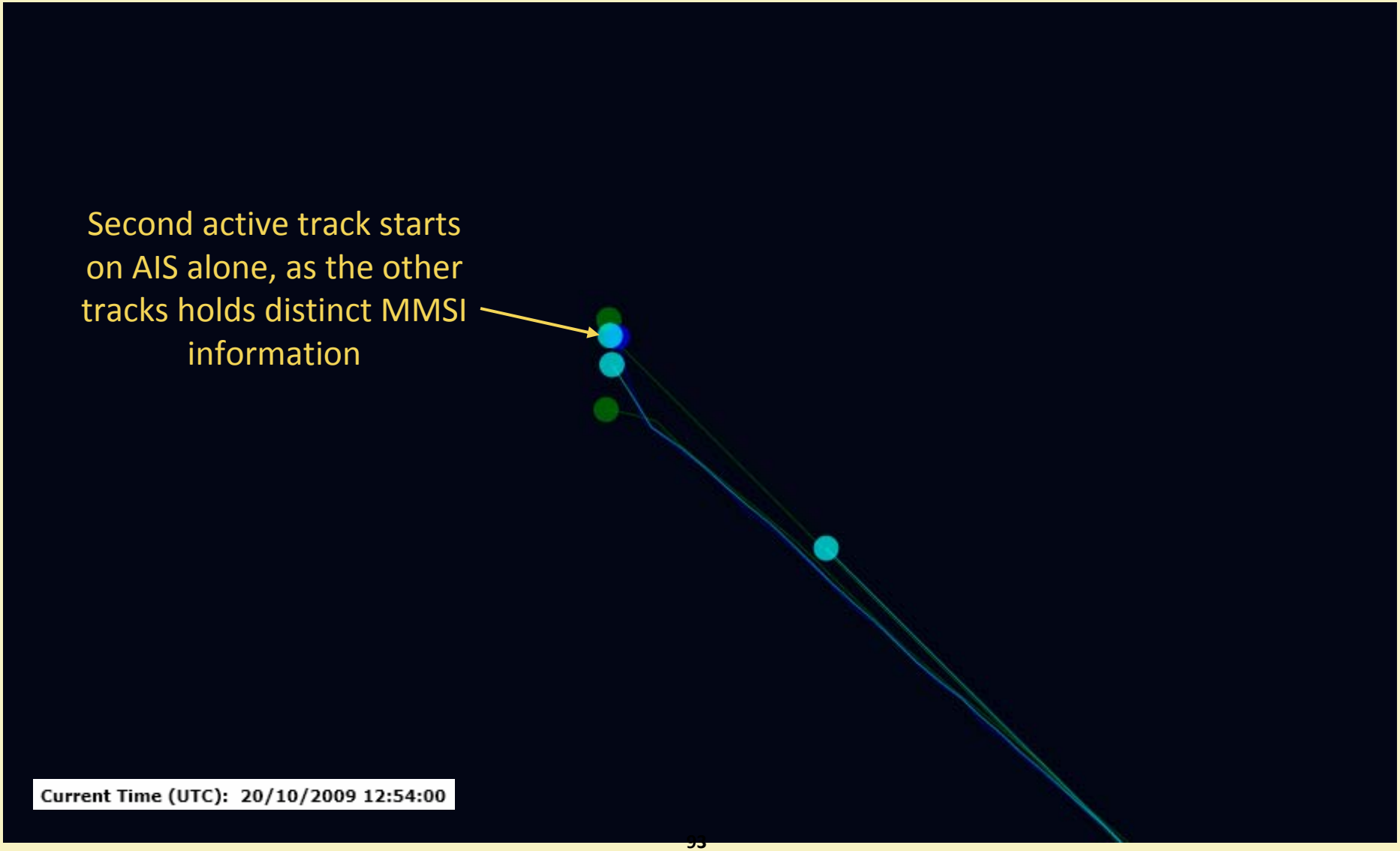
AIS Info:

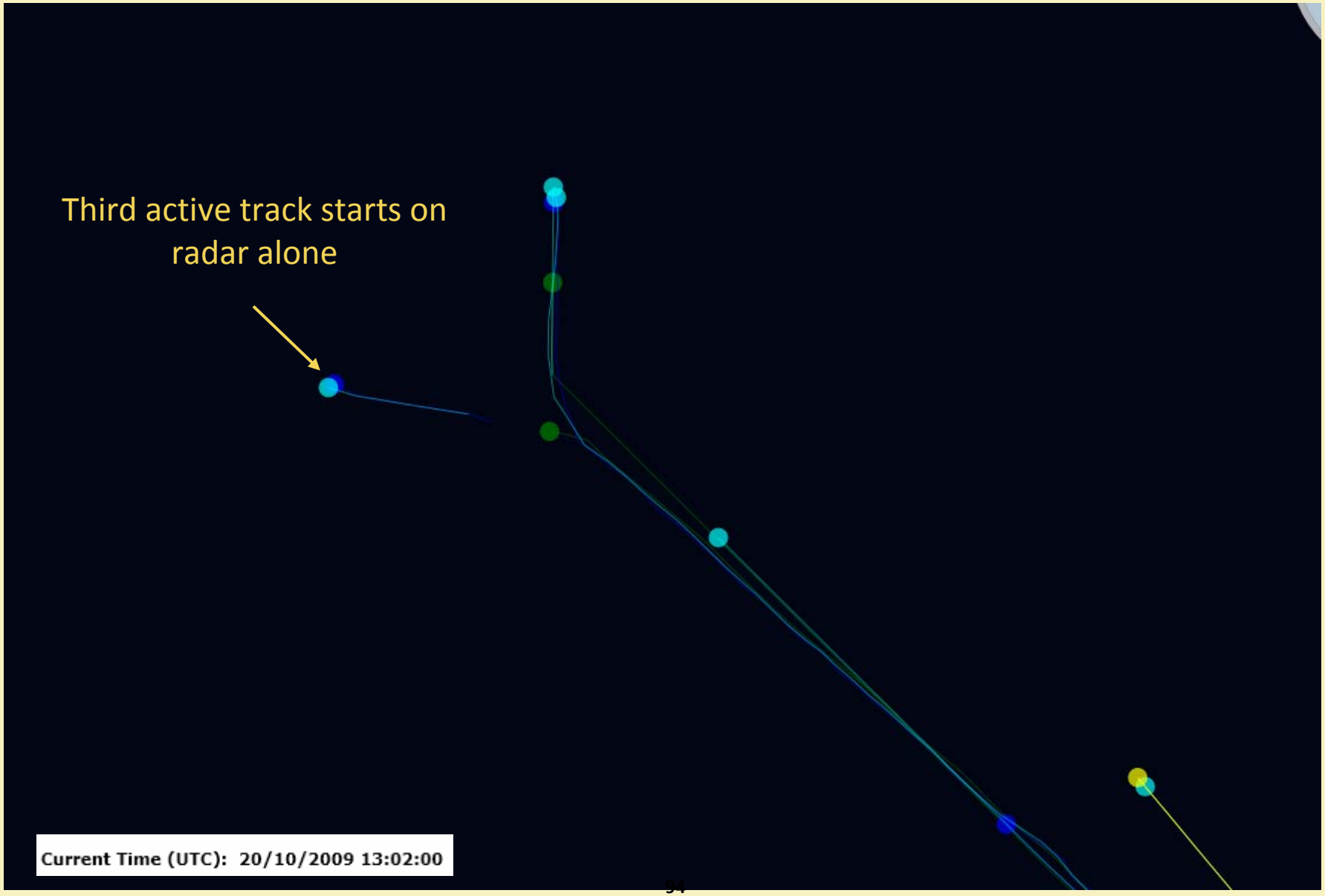
Name	MMSI	IMO	Type	Cargo	Destination
EUROCARGO VALENCIA	247579000	9192959	7-Cargo	9-NoInfo	ASHDOD

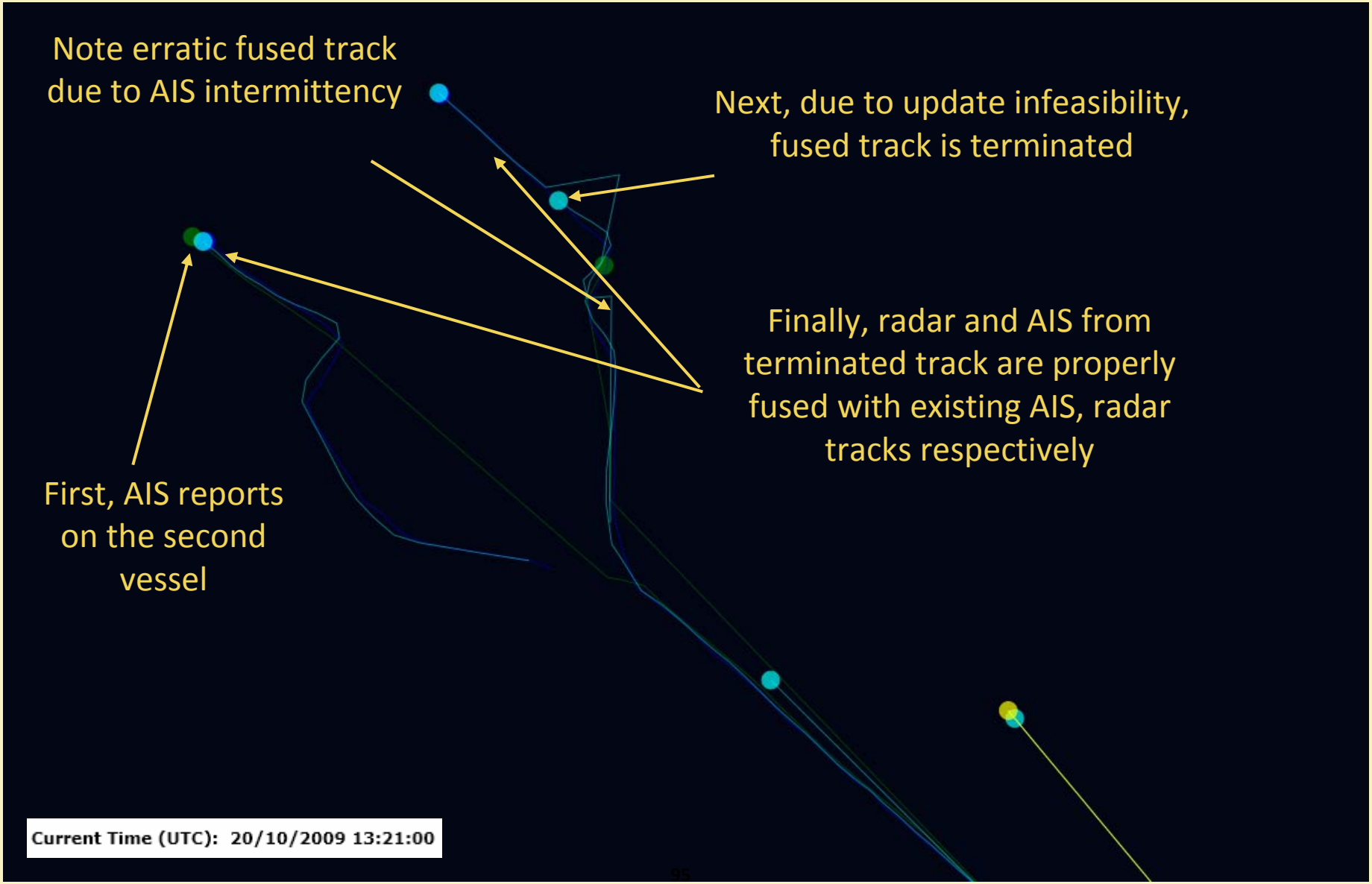


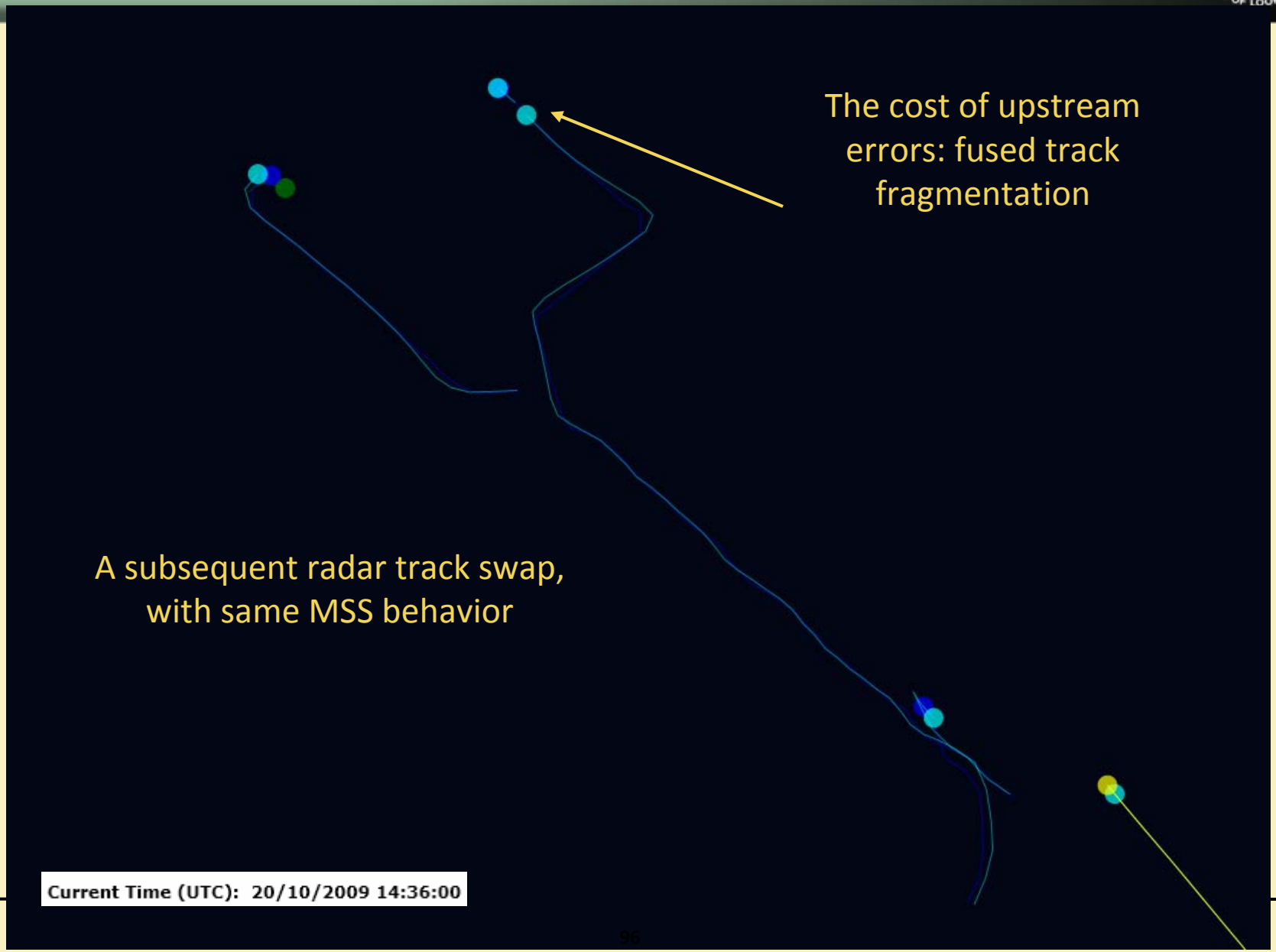

Fused track termination
Last AIS transmission
Dropped radar track
Current Time (UTC): 20/10/2009 12:49:00


Maneuvering targets begin
to expose radar track swap
Current Time (UTC): 20/10/2009 12:53:00

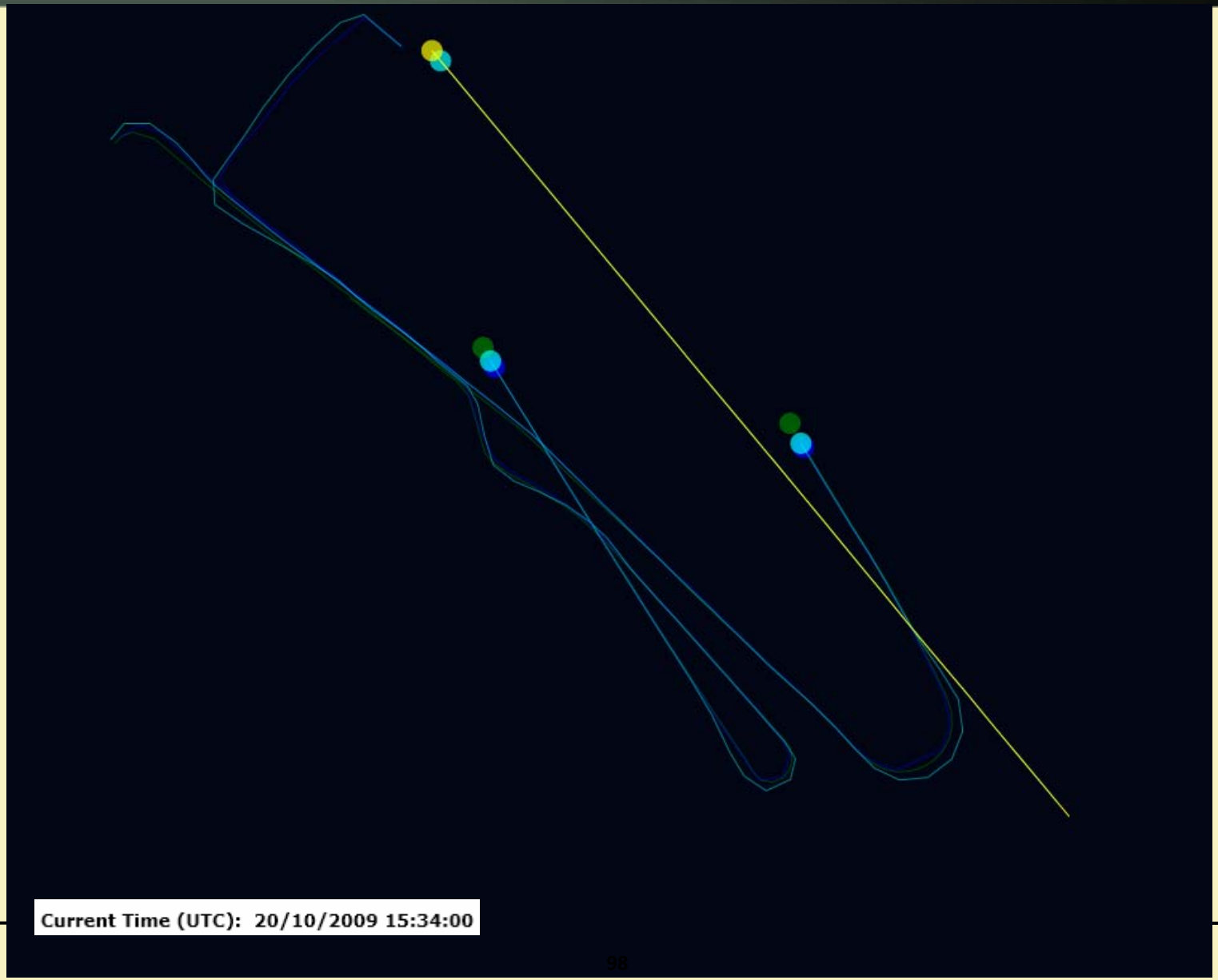

Second active track starts on AIS alone, as the other tracks holds distinct MMSI information
Current Time (UTC): 20/10/2009 12:54:00


Third active track starts on radar alone
Current Time (UTC): 20/10/2009 13:02:00

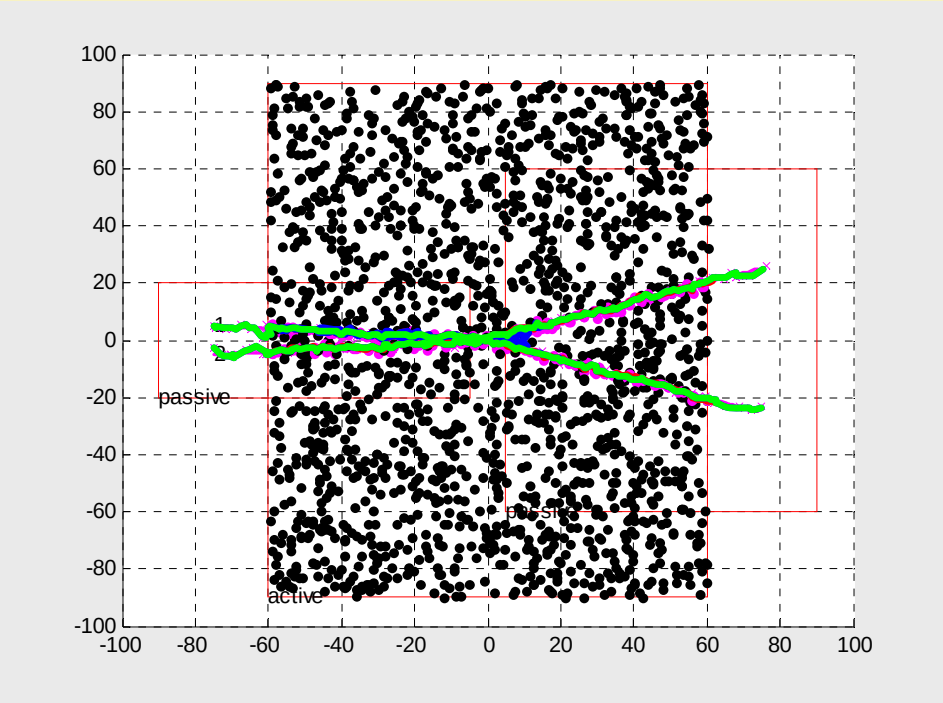

Note erratic fused track due to AIS intermittency
Next, due to update infeasibility, fused track is terminated
Finally, radar and AIS from terminated track are properly fused with existing AIS, radar tracks respectively
First, AIS reports on the second vessel
Current Time (UTC): 20/10/2009 13:21:00


The cost of upstream errors: fused track fragmentation
A subsequent radar track swap, with same MSS behavior
Current Time (UTC): 20/10/2009 14:36:00

Current Time (UTC): 20/10/2009 15:18:00


Current Time (UTC): 20/10/2009 15:34:00

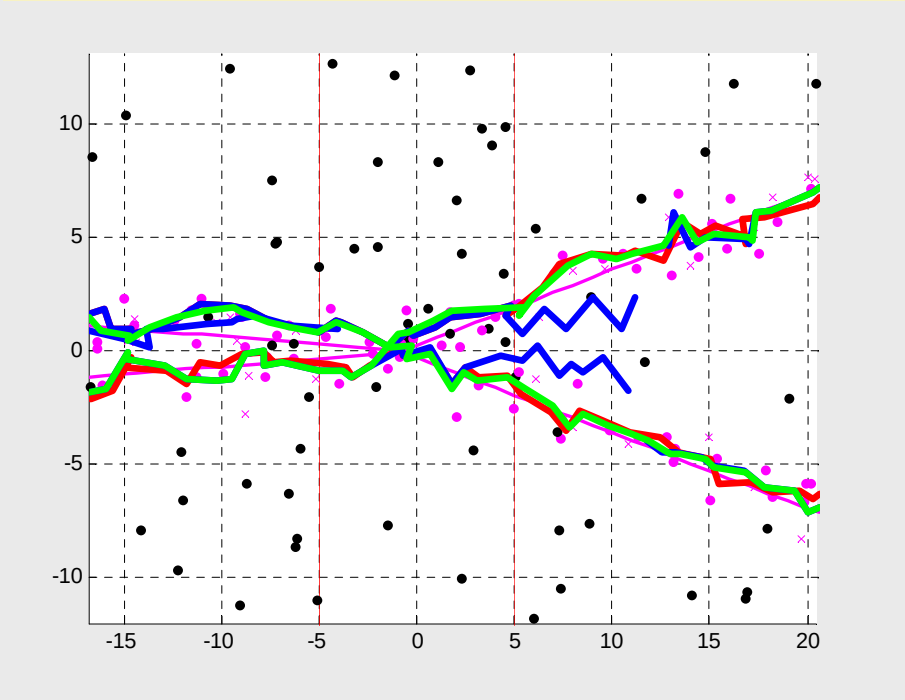
ENHANCED TRACK CONTINUITY: TRACK-BREAK-FUSE



Track

Track-before-fuse

Track-break-fuse

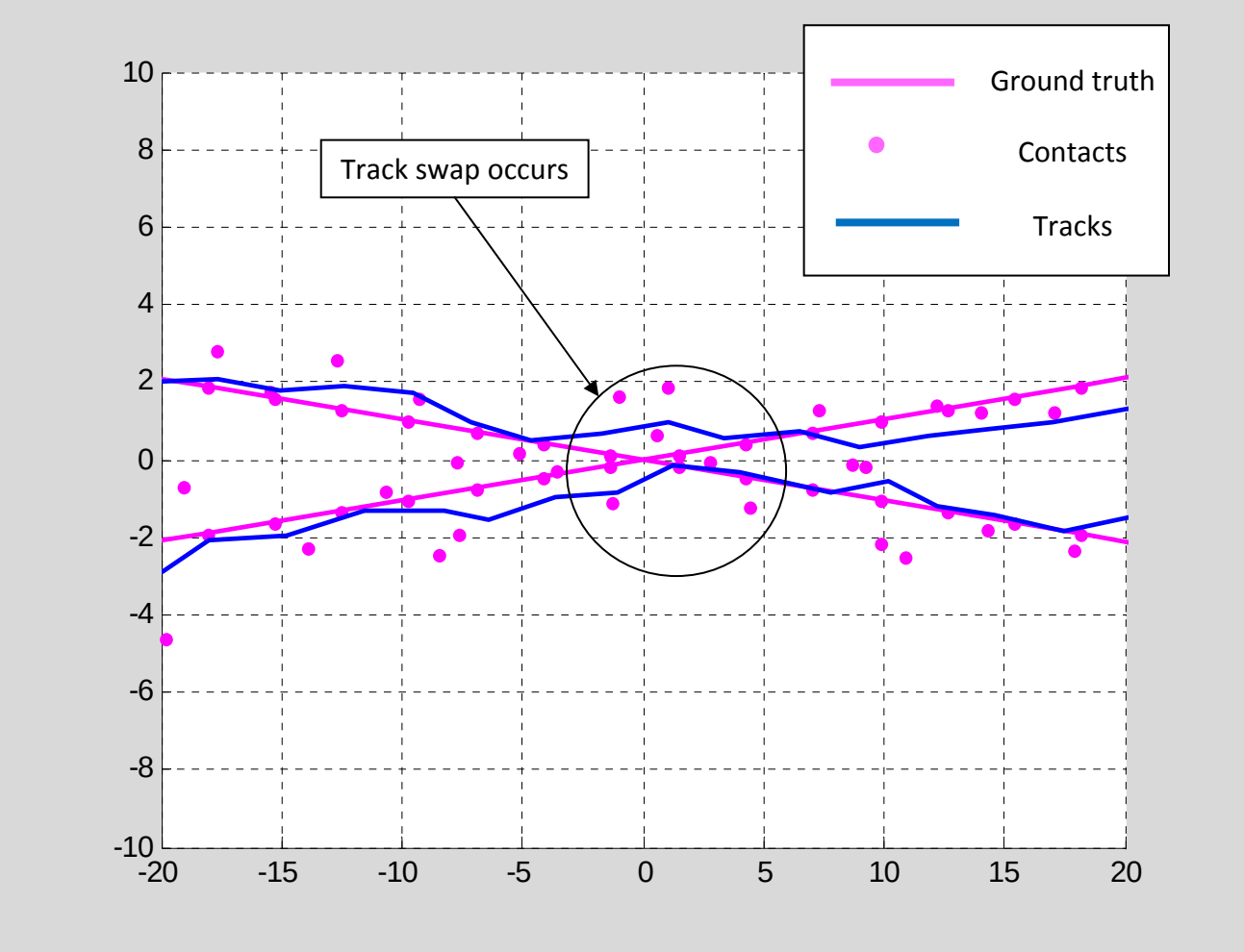


- Track-while-fuse is limited to small *n-scan*

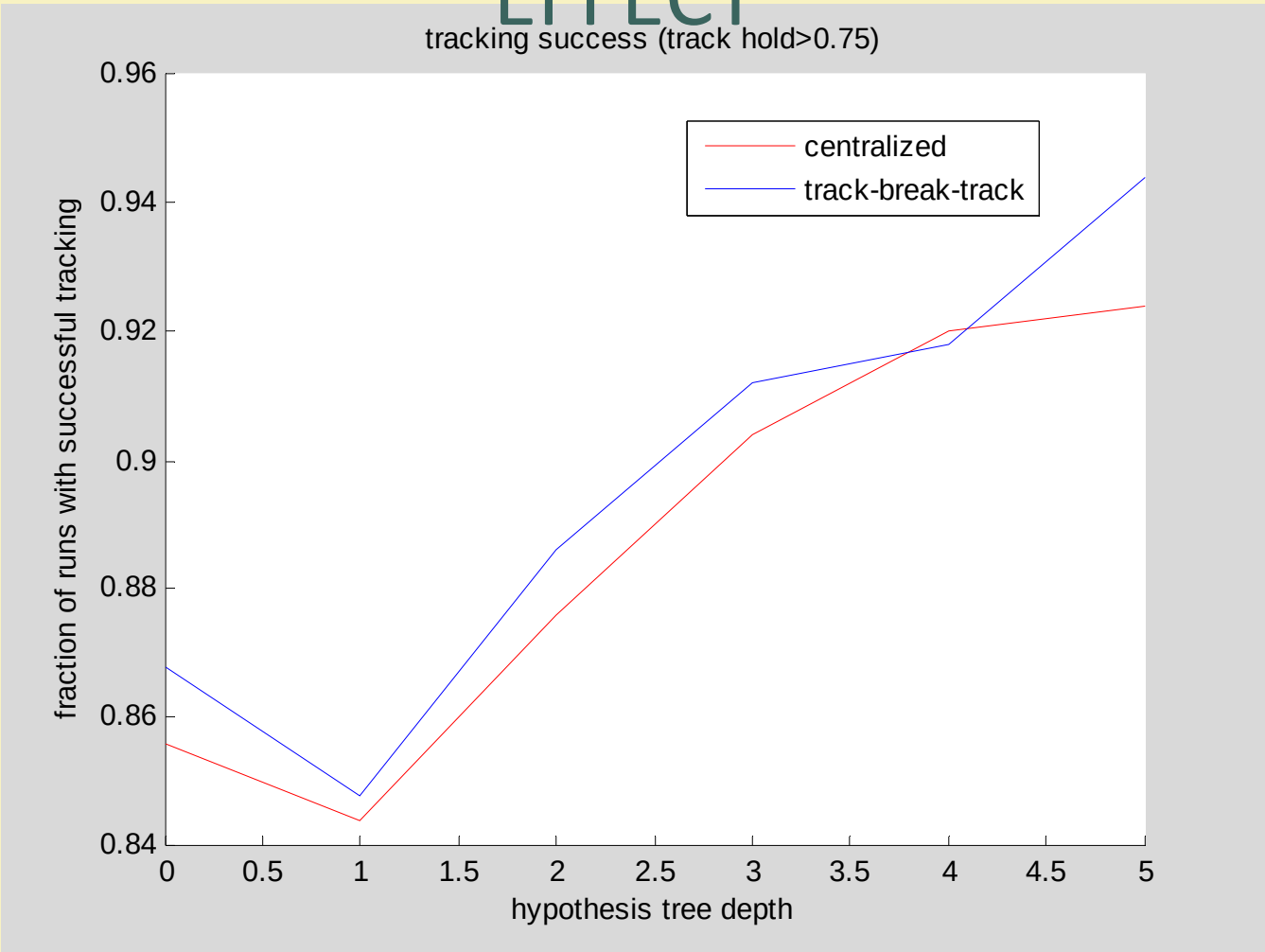
PERSISTENT MULTI-TARGET AMBIGUITIES

- Multi-scale data: *track-before-fuse* & *track-break-fuse*
- **Slowly crossing targets: *track-break-fuse***
- Large sensor networks: *fuse-before-track*
- Group tracking: *track-segment-fuse*

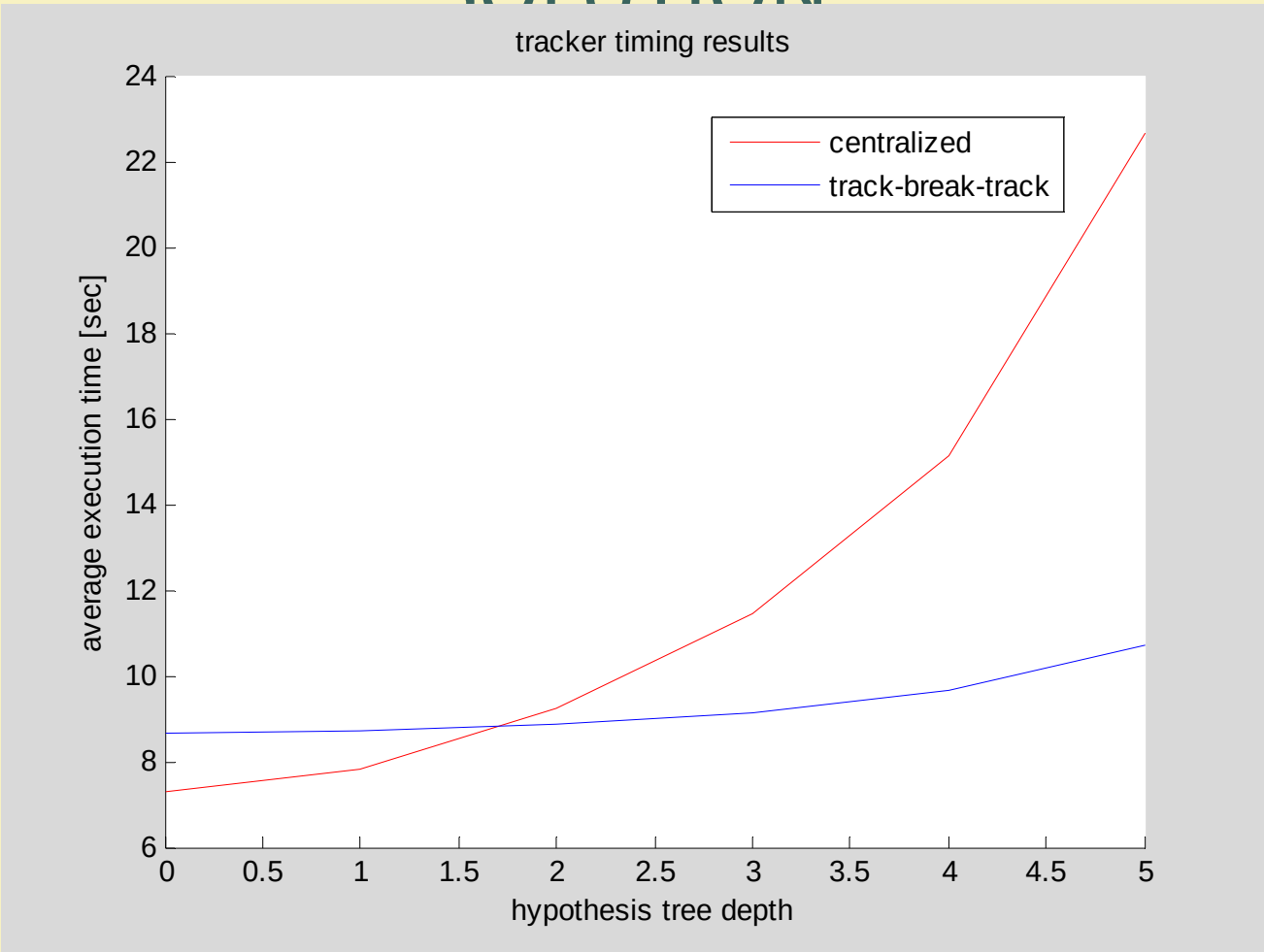
TRACK REPULSION EFFECT



COMBATING THE TRACK REPULSION EFFECT



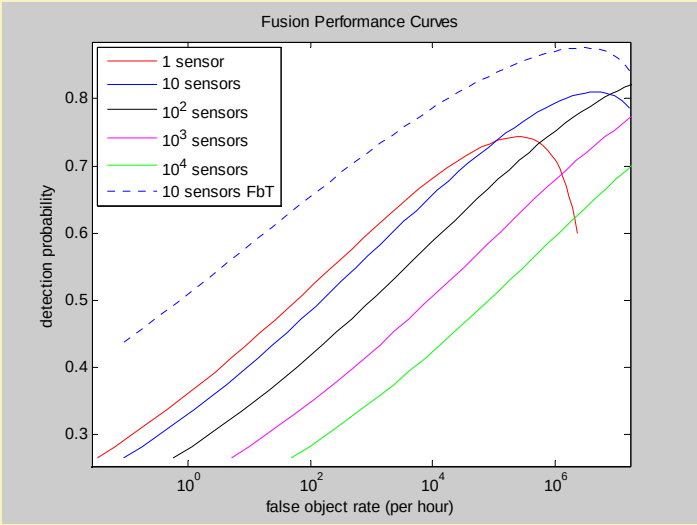
TRACK REPULSION EFFECT: A SCALABLE SOLUTION

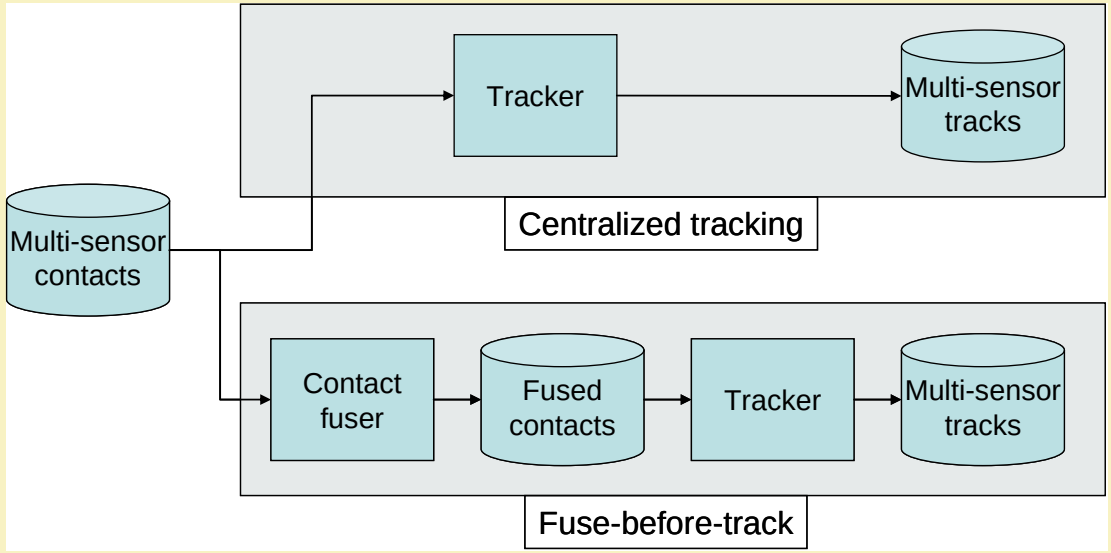


PERSISTENT MULTI-TARGET AMBIGUITIES

- Multi-scale data: *track-before-fuse* & *track-break-fuse*
- Slowly crossing targets: *track-break-fuse*
- **Large sensor networks: *fuse-before-track***
- Group tracking: *track-segment-fuse*

MODEL-BASED RESULTS



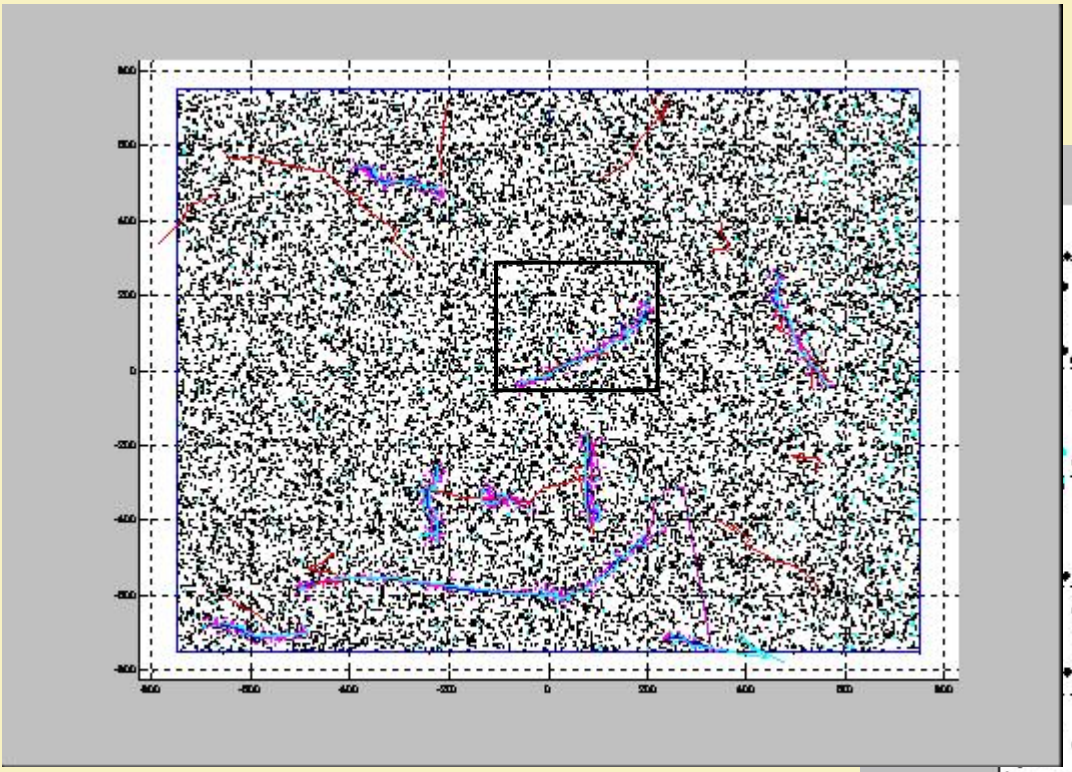


compunetix

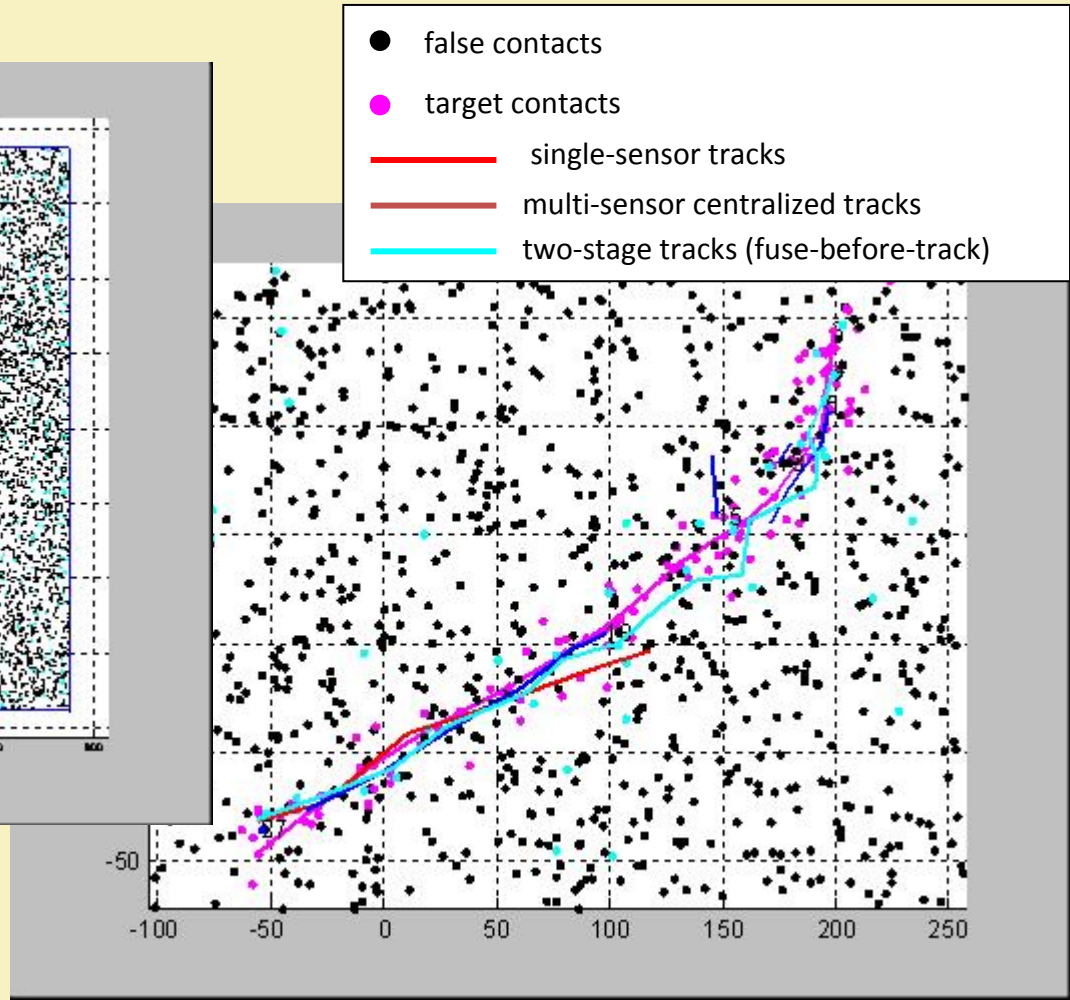
LARGE SENSOR NETW

FUSE-BEFORE-TRACK

Metric	Single-sensor	Multi-sensor	FbT
Track hold	0.092	0.473	**0.659**
False track rate	288.7	171.2	**65.0**
Execution time	21.9sec	171.6sec	**8.53sec**



In ten-sensor study, FbT is best



ML-MHT, MHT^2, AND ML-MHT^2

- ML-MHT
  - Reduce contact data via ML processing on simultaneous sensor scans

$$\Lambda(X) = \prod_{i=1}^{N} \left\{ \frac{1-p}{u^{n_i}} \mu_\lambda(n_i) + \frac{p \cdot \mu_\lambda(n_i - 1)}{u^{n_i - 1} n_i} \sum_{j=1}^{n_i} p(Z_{ij} \mid X) \right\},$$

$$Z = X + w \qquad w \sim N(0, \Sigma)$$

$$\mu_\lambda(n) = \frac{\lambda^n \exp(-\lambda)}{n!}$$

$$p(Z \mid X) = \frac{1}{2\pi |\Sigma|^{0.5}} \exp\left(-\frac{1}{2} (Z - X)' \Sigma^{-1} (Z - X) \right)$$

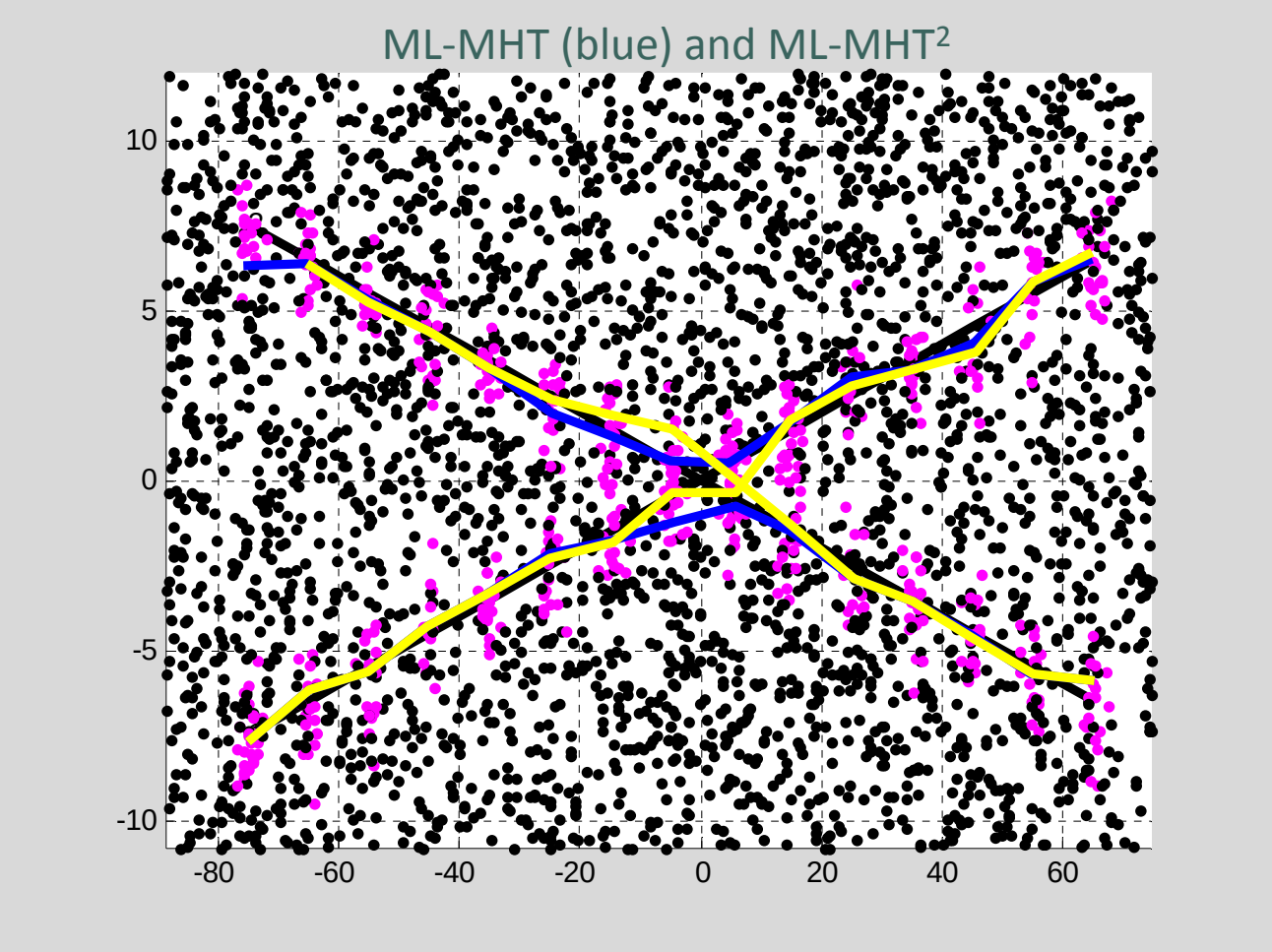
  - Evaluate likelihood surface only at contact locations and choose top contacts
  - No collapsing of sensor data into one fused scan
  - Reduced dataset provided to MHT

$$M > p \cdot \lambda_T \cdot N$$

- ML-MHT^2 and MHT^2
  - First-stage MHT: track break at non-zero time steps
  - ML-MHT^2 includes as well the ML processing front end

ML-MHT PROCESSING (FUSE-BEFORE-TRACK)

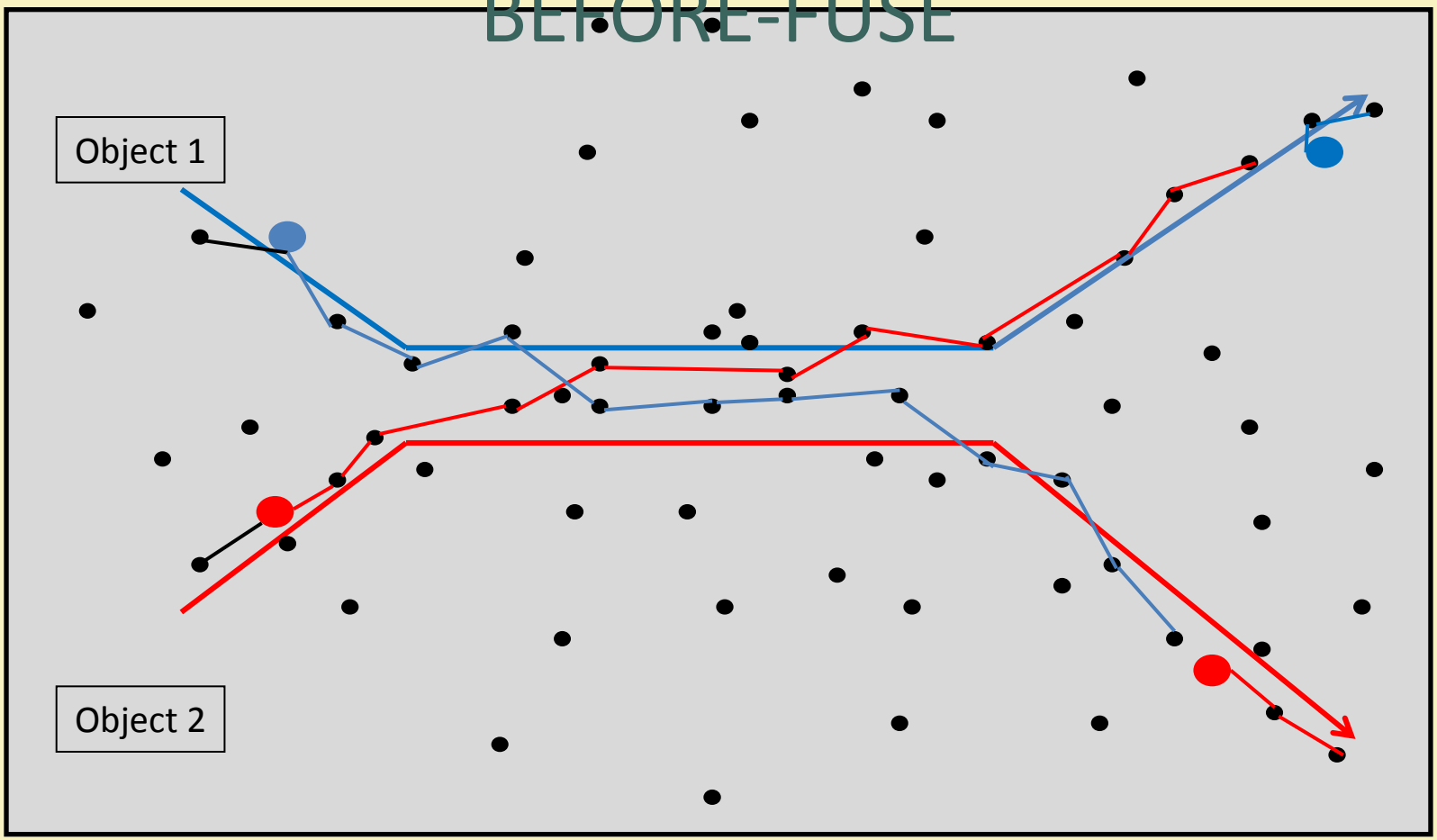
40-sensor example



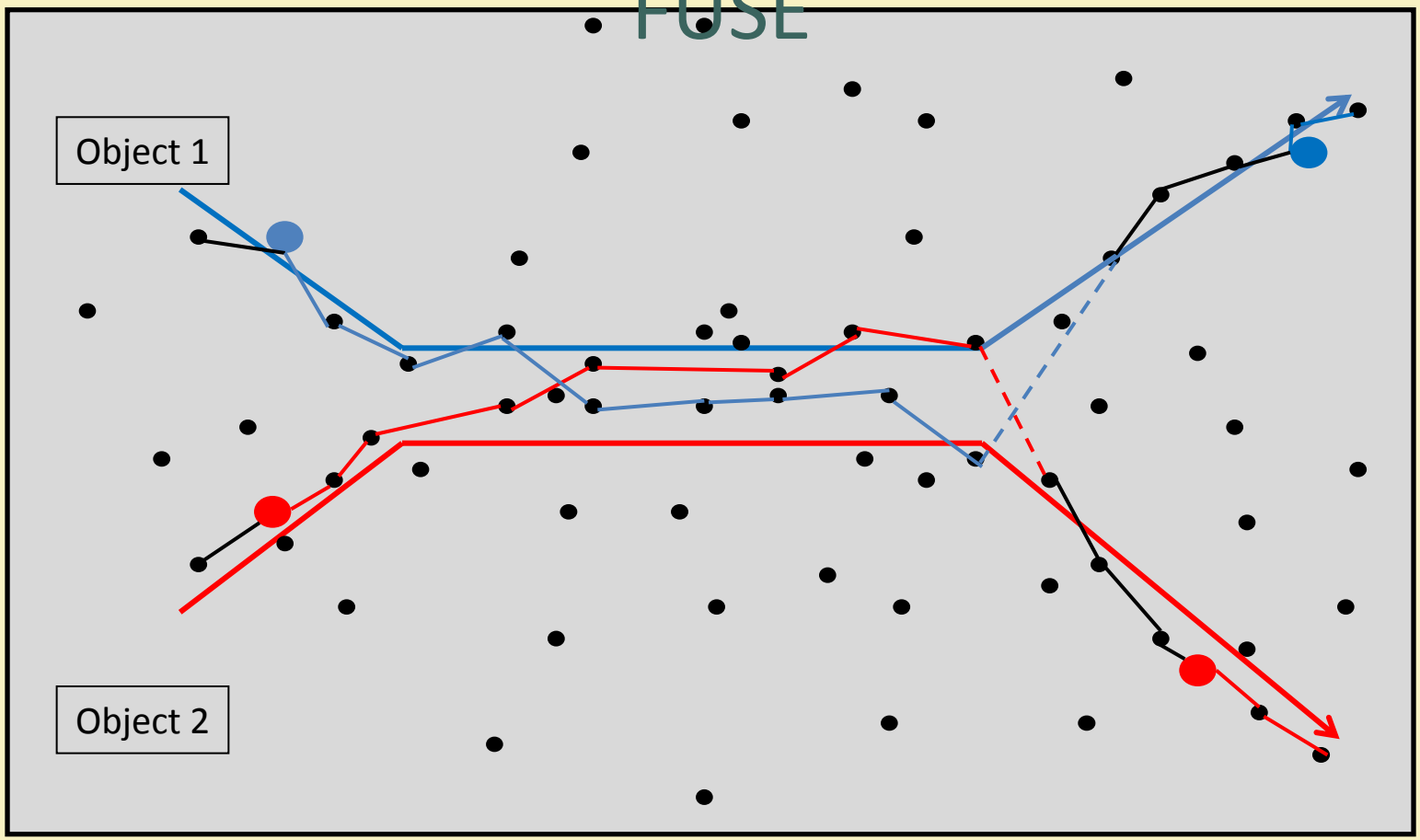
PERSISTENT MULTI-TARGET AMBIGUITIES

- Multi-scale data: *track-before-fuse* & *track-break-fuse*
- Slowly crossing targets: *track-break-fuse*
- Large sensor networks: *fuse-before-track*
- **Group tracking: *track-segment-fuse***

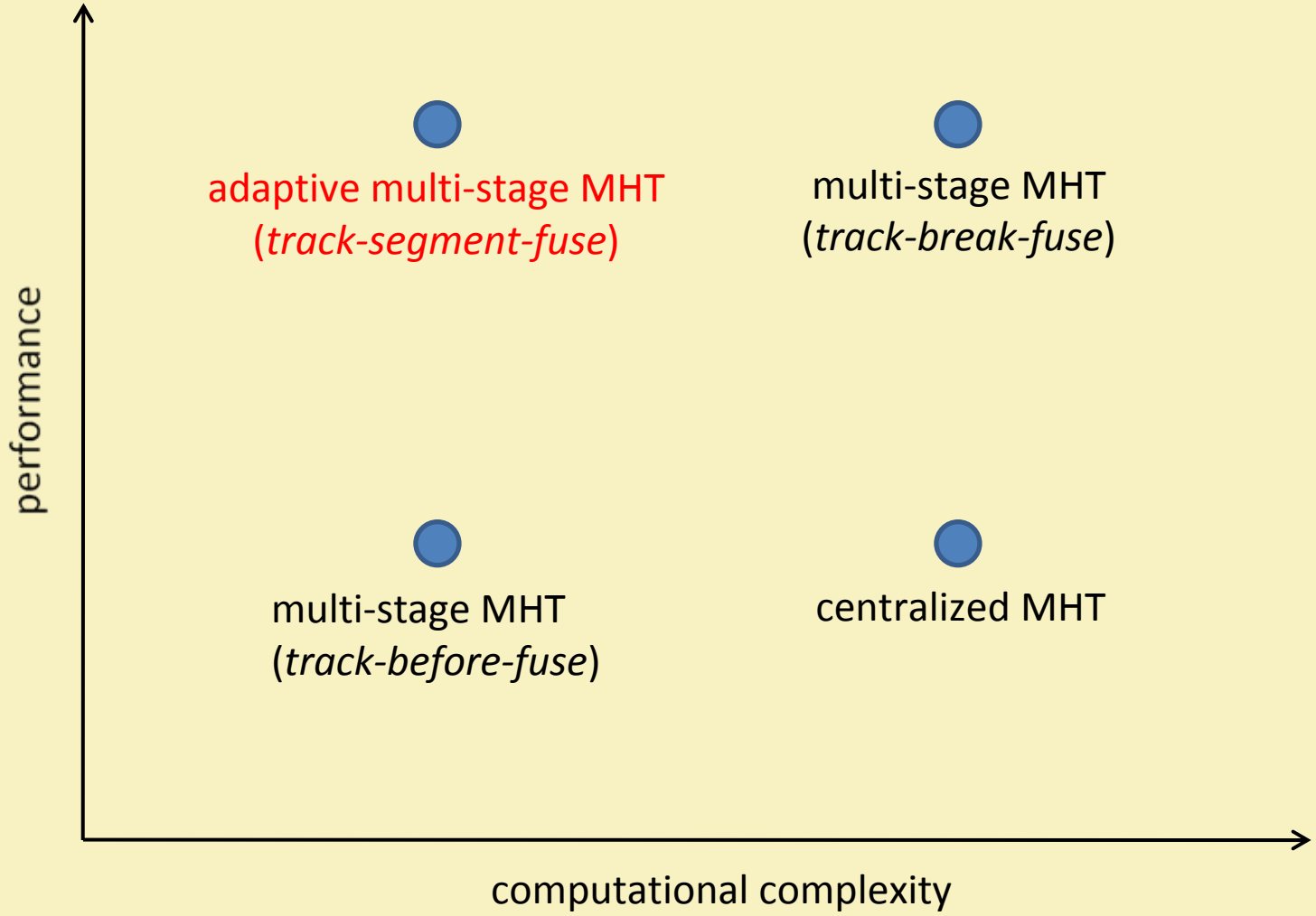
REVISITING THE PROBLEM IN TRACK-BEFORE-FUSE



A POSSIBLE SOLUTION: TRACK-SEGMENT-FUSE



ADAPTIVE MULTI-STAGE MHT



CONCLUSIONS

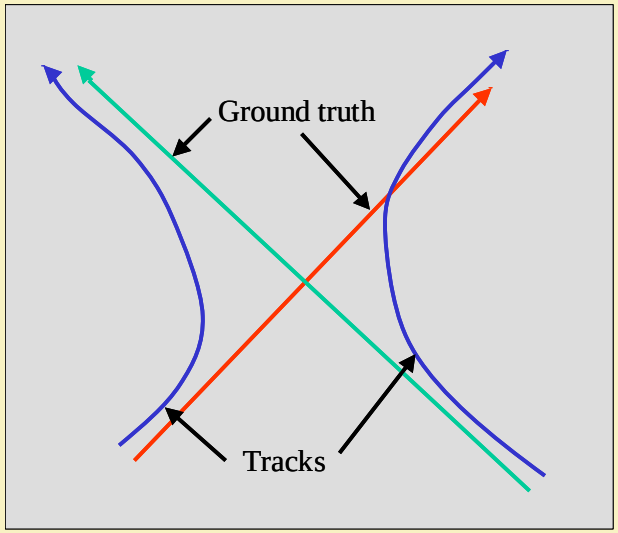
- Effective data fusion is crucial to many surveillance problems in security and defense
- Multi-hypothesis (MHT) and multi-stage MHT provide a powerful approach in difficult surveillance settings
  - Key challenge is hypothesis management
  - Identifying a single hypothesis, even if it is the true MAP solution, is potentially problematic in some settings
- Some open research directions
  - Hybrid approaches that combine strengths of MHT with other approaches, e.g. batch processing (ML), label-free tracking (SJPDA, CPHD)
  - Equivalence-class MHT (EC-MHT) for dense-target scenarios
- For some surveillance problems, physical-space reasoning is insufficient
  - Additional data sources and higher-level processing are required

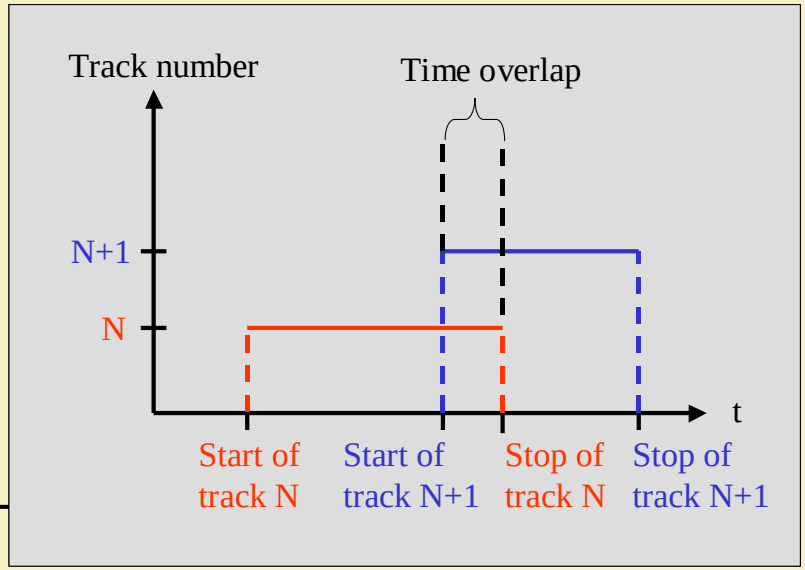
OUTLINE

- Target modeling
- Bayesian filtering
- Multi-target filtering
- Multi-target tracking
- Cardinality tracking
- Distributed tracking
- **Other topics**

PERFORMANCE EVALUATION

- Metrics
  - Track PD, track FAR
  - Fragmentation
  - Localization error
- Issues
  - Track-truth assignment
  - Common waypoint determination
  - Fixed clutter tracks
  - Average or worst case latency
  - Variations (FAR to reflect track duration; velocity accuracy)
  - Other metrics (time-to-detect, probability of correct association, track purity, track consistency)





APPLICATION TO MARINE MAMMALS

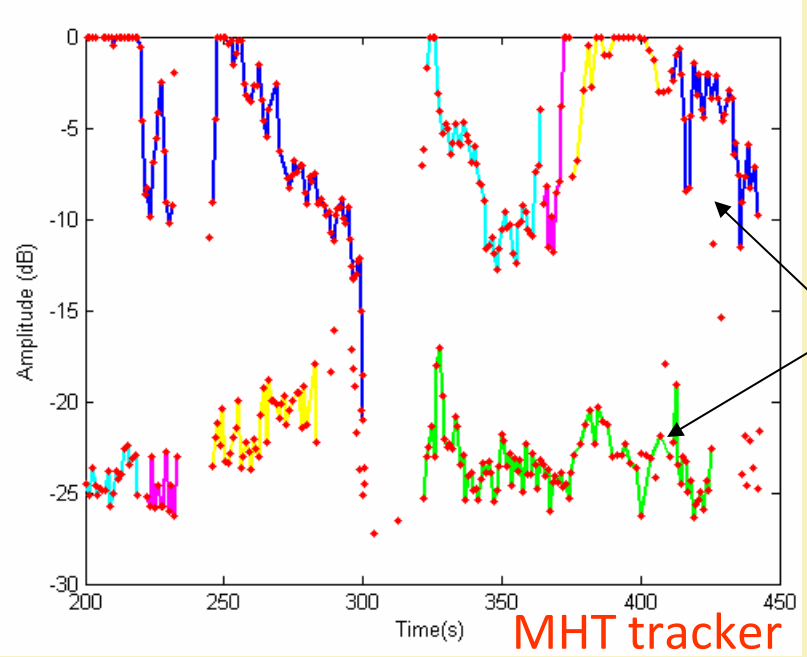
- Problem statement
  - Estimate the number of whales in a surveillance region
- Assumptions
  - Click amplitude and Inter-Click Interval (ICI) are slowly varying
- Dynamical model

$X_k = X_{k+1} + w_k$ $\quad$ w_k with variance $q_w\,(t_k - t_{k-1})$

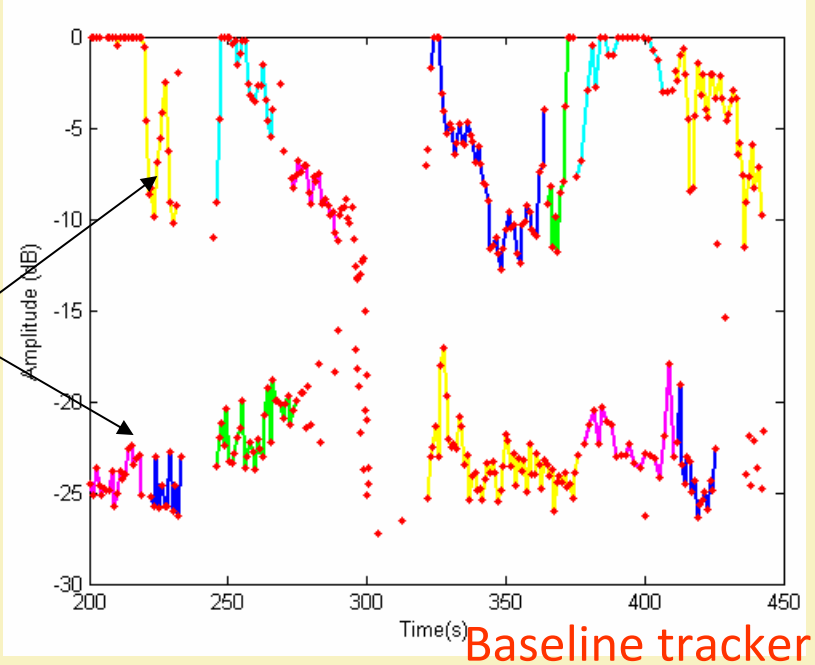
$(t_{k+1} - t_k) = (t_k - t_{k-1}) + v_k$ $\quad$ v_k with variance $q_v\,(t_k - t_{k-1})$

- Measurement model
  - Precise measurement of click amplitude and time
- Algorithm determines a lower-bound on mammal presence
  - the same mammal may generate multiple vocalization sequences interrupted by periods of silence

SOME PRELIMINARY RESULTS



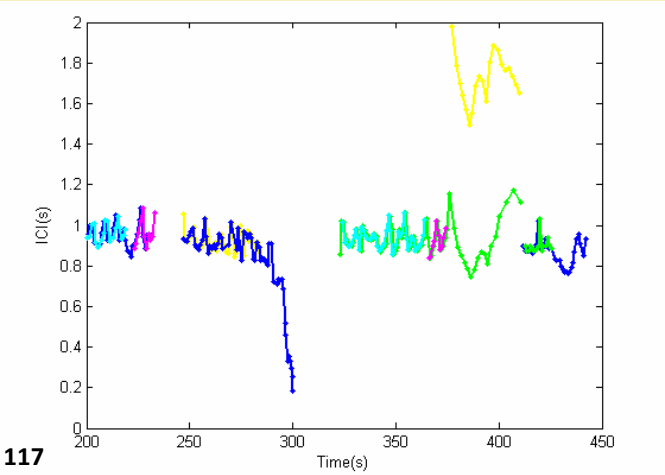
2 animals
or
only 1 animal
(and echo)
?



Click amplitude sequence (in red), and the resulting automatically-originated tracks

ICI

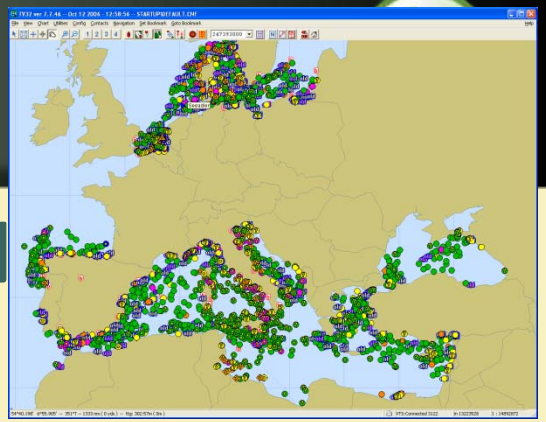
1 animal and its echo

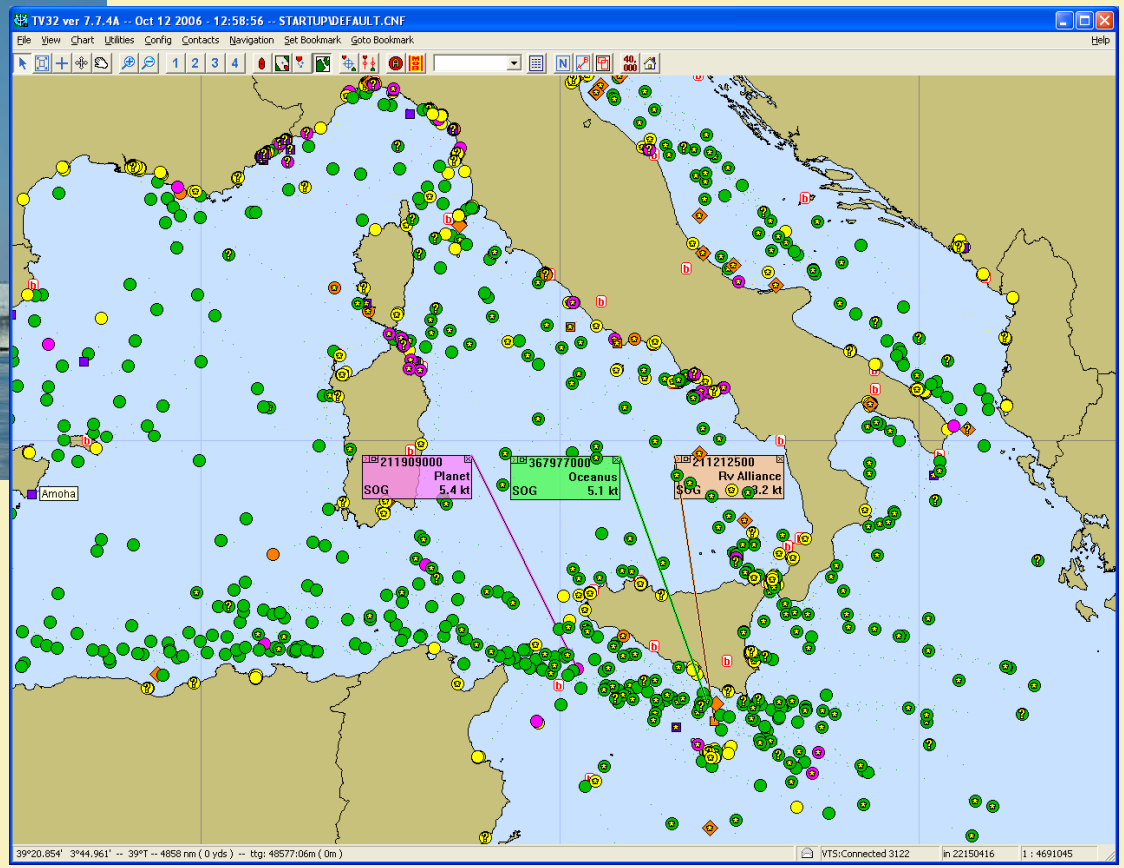


compunetix

MARITIME SURVEILLAN

- Fusion of active sonar, AIS, radar, SAR
- Anomaly detection



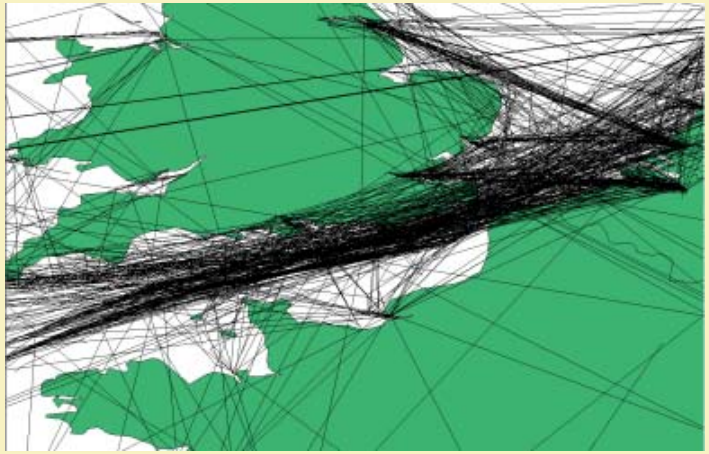


AIS DATA ISSUES

- Maritime picture is cluttered with AIS tracks

  ⇒Need to filter useful information due to:

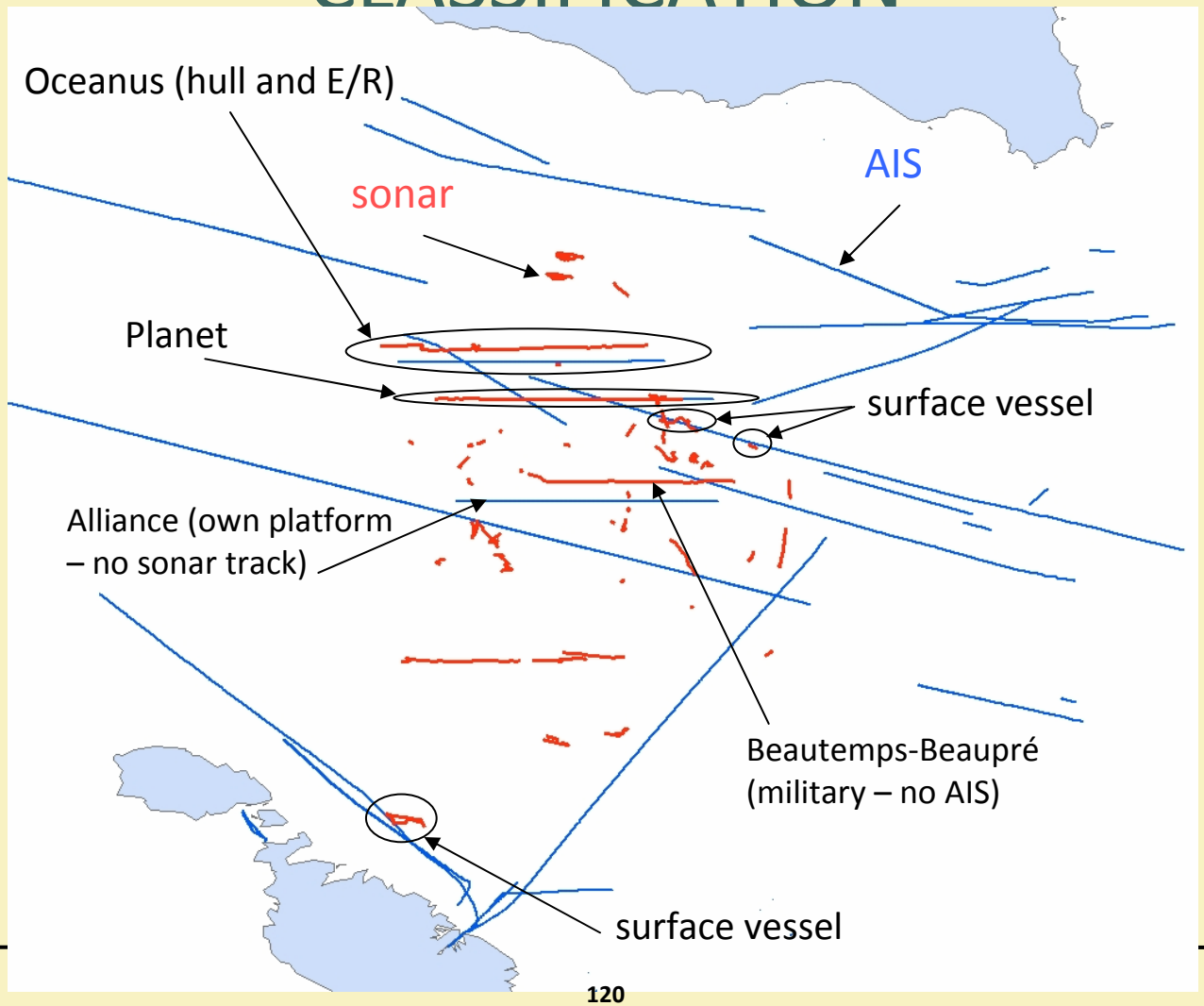
  - Limited resources to analyze / investigate potentially 4 million AIS messages a day => need for automated systems
  - large number of problems in AIS messages: Manually input errors, Wrong settings, some 42% of all AIS messages showed at least 1 problem, out of range in Lat – Lon fields of AIS found to be as high as 1%

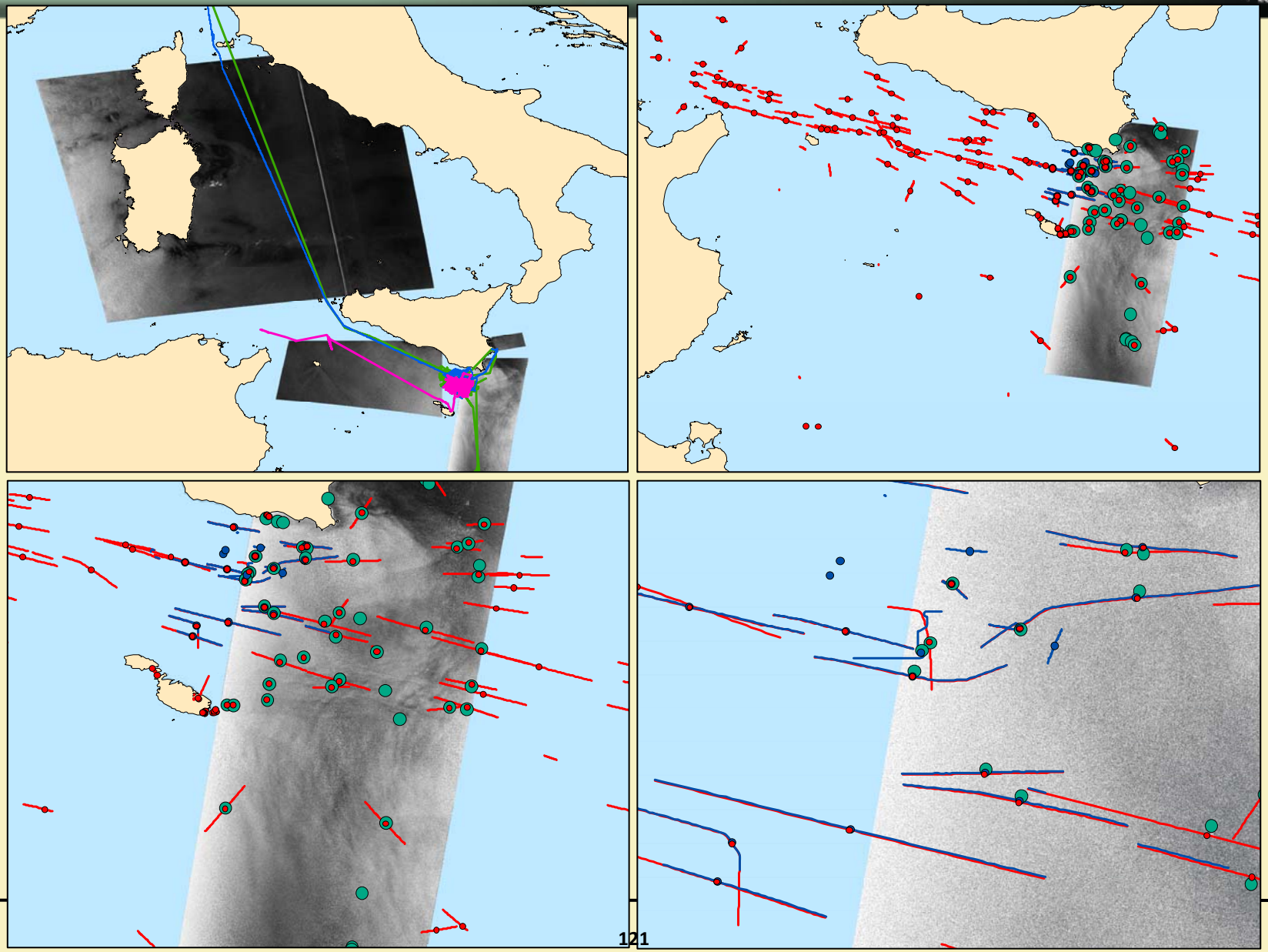


Corrupted AIS data

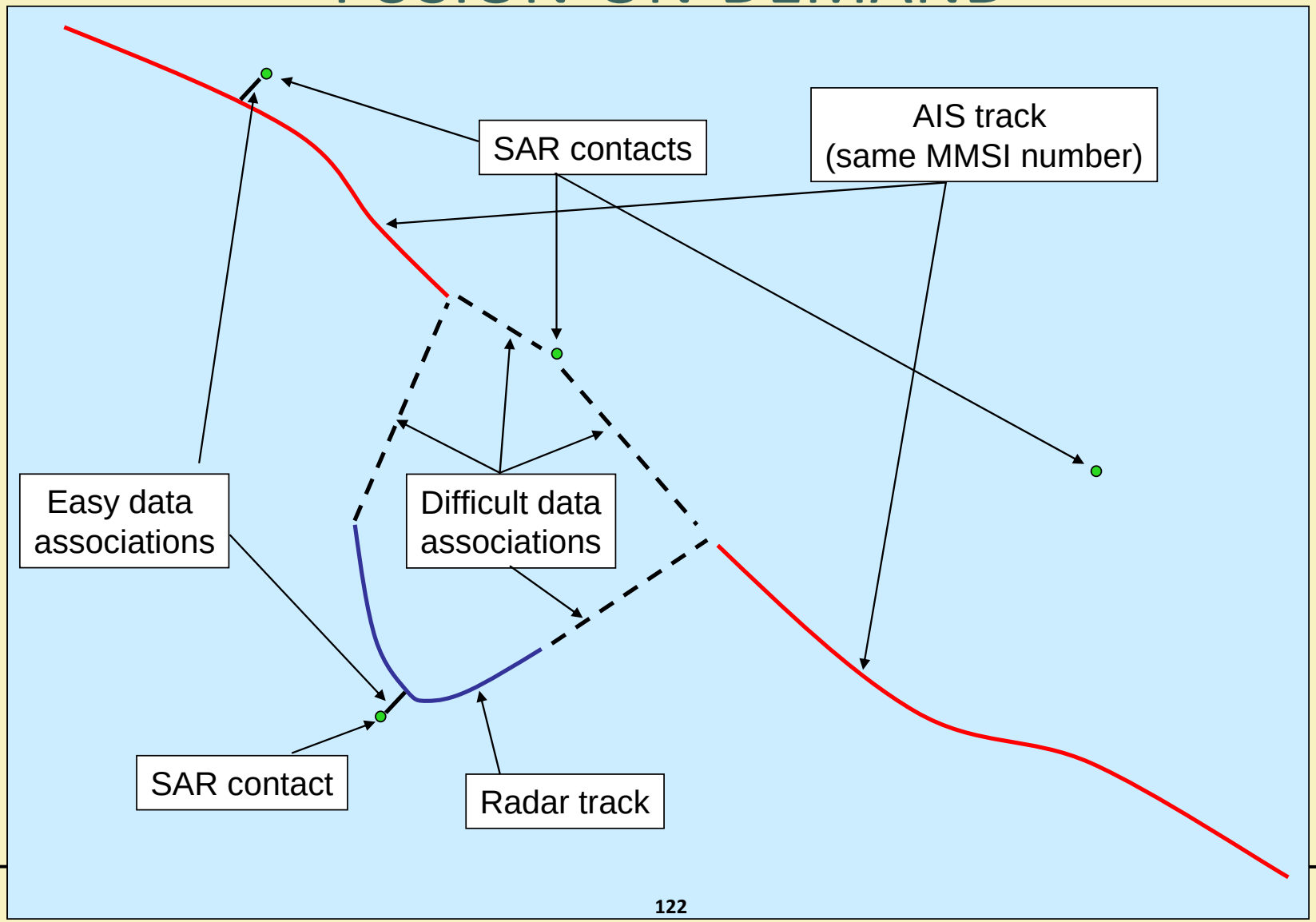
Processed AIS data

AIS SUPPORT TO ACTIVE SONAR TRACK CLASSIFICATION

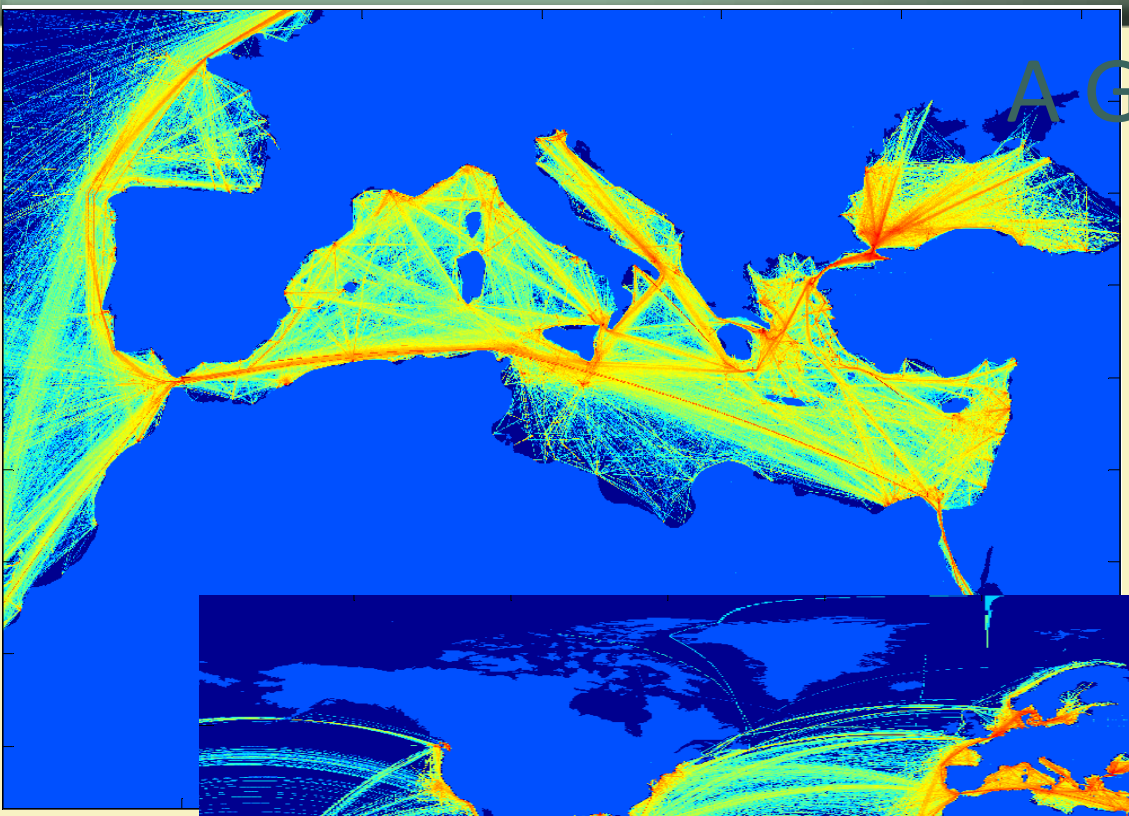




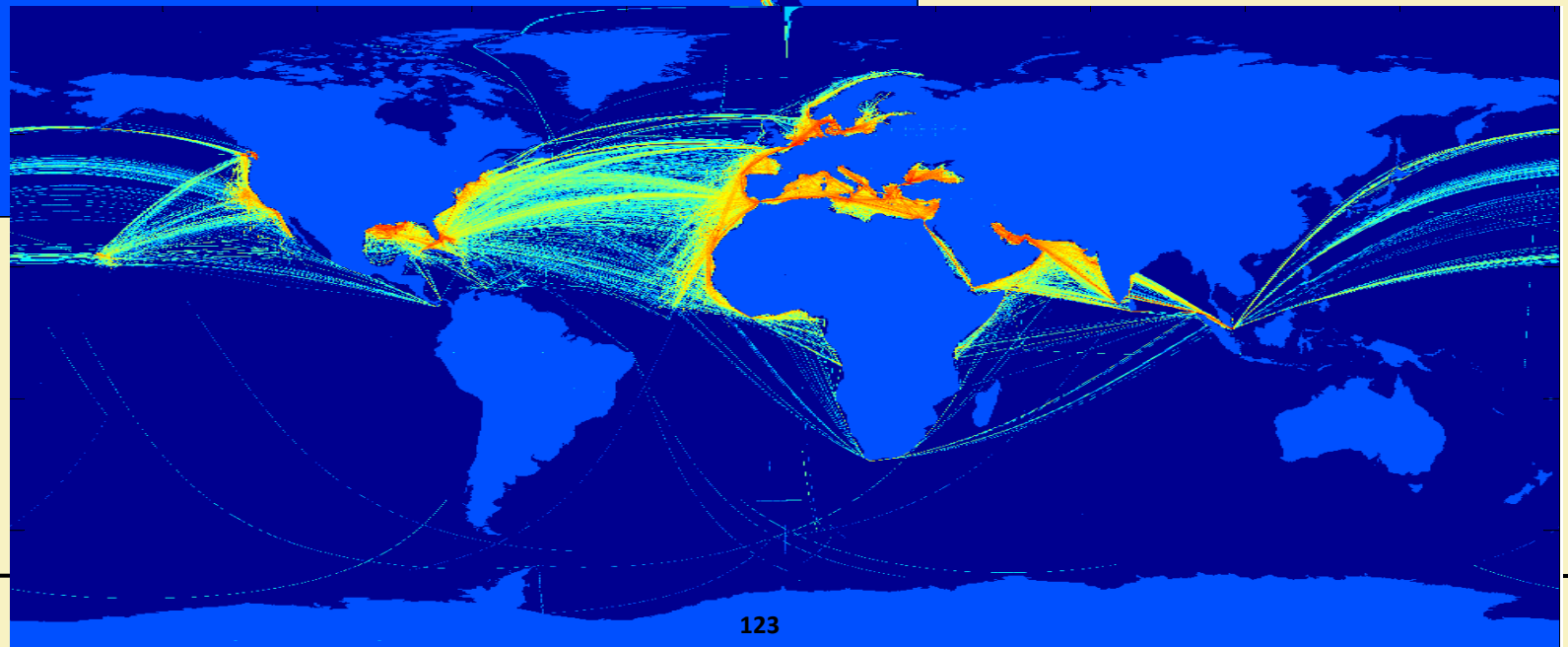
FUSION-ON-DEMAND



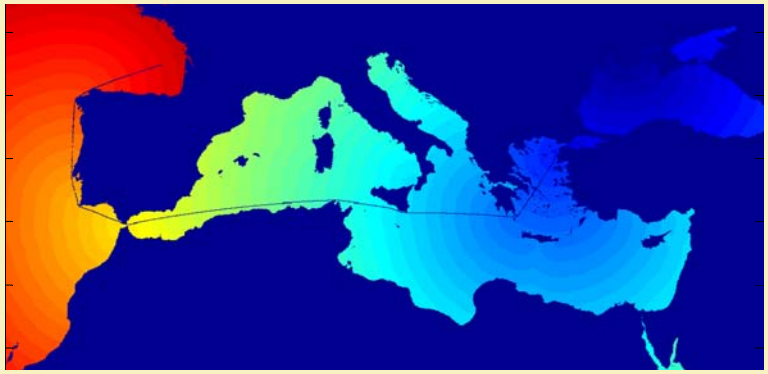
A GLOBAL PROBLEM

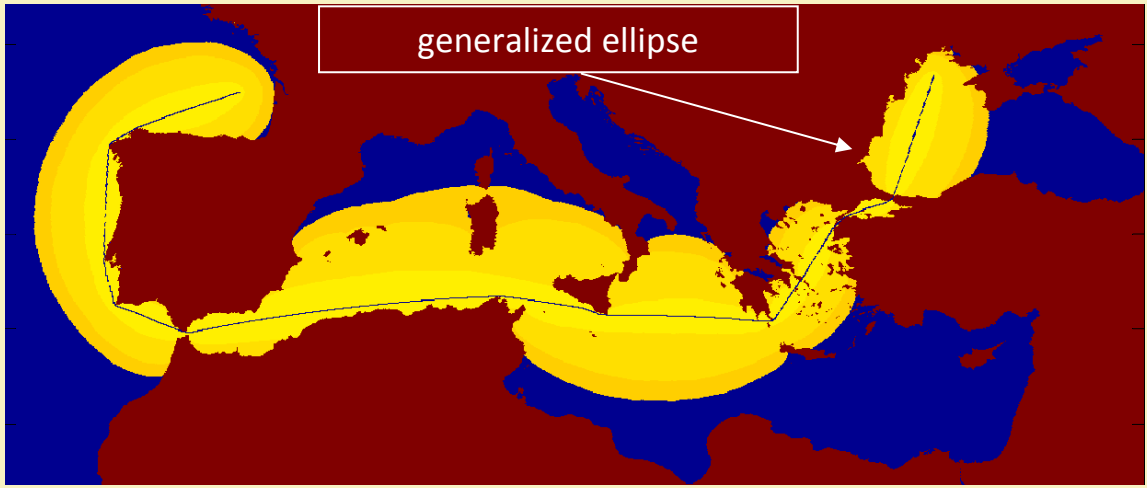


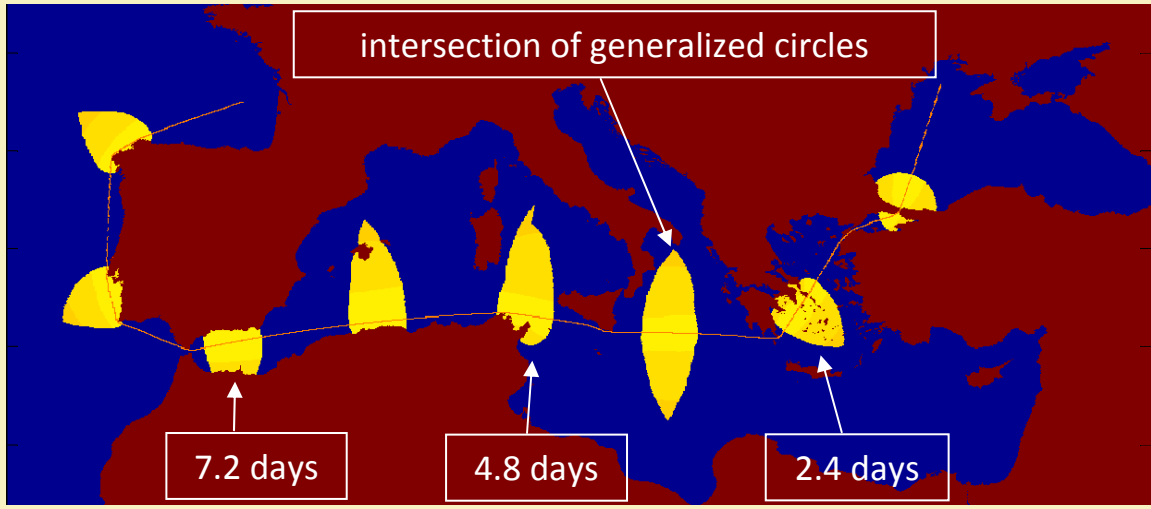
- Maritime Safety & Security Information System (MSSIS) Data
- Histogram representation of 6 months of MSSIS data along with hypothesized shortest distance tracks



FUSION-ON-DEMAND CONCEPT



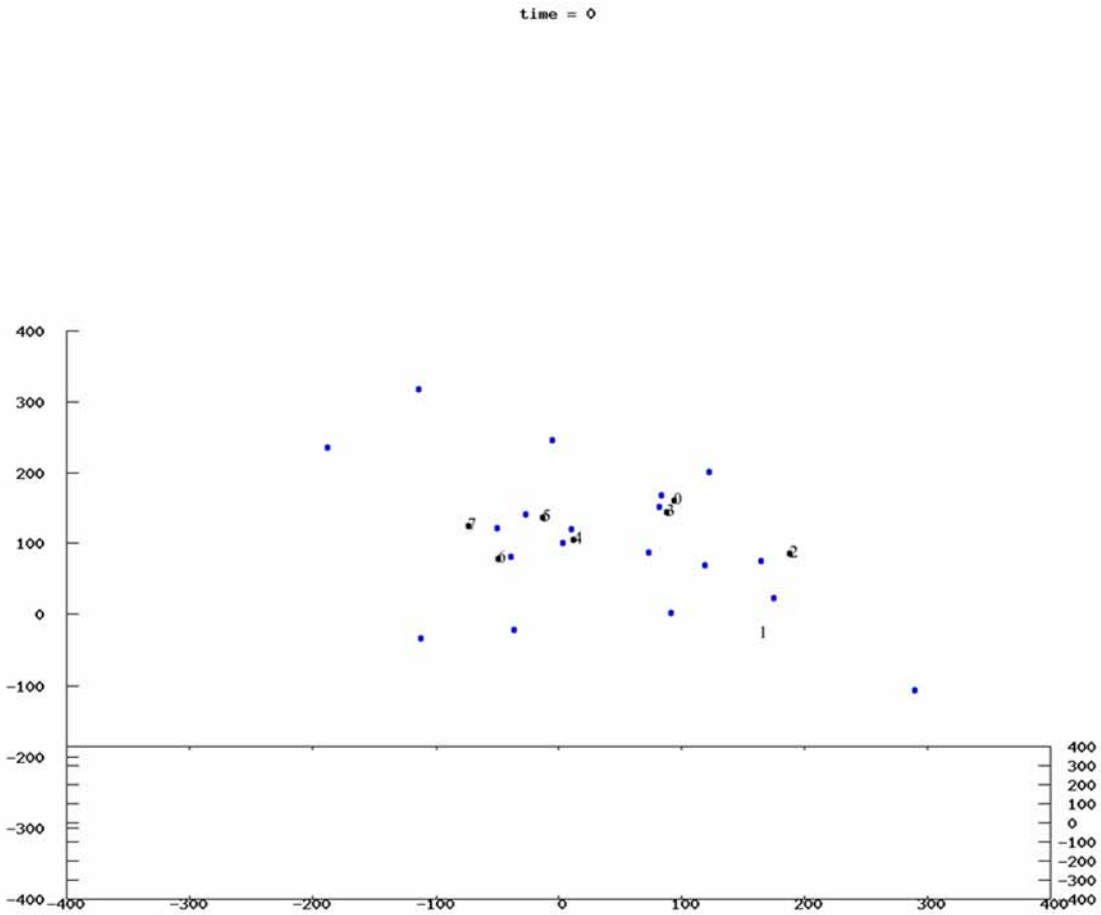




VIDEO SURVEILLANCE

- Some issues
  - Image plane tracking vs. 3D tracking
  - Single fixed sensor vs. multiple moving sensors
  - Object-level tracking vs. pixel-level tracking
  - Sensor modeling (partial obscurations, merged measurements)
  - Feature-aided tracking

fragmentation resulting from merged measurements

VIDEO TRACKING

TRACK VISUALIZATION